CHAPTER-1

PROPERTIES OF FLUID

1.1 Define fluid

1.2 Description of fluid properties like Density, specific weight, specific gravity, specific volume and solve simple problems.

1.3 Definitions and Units of Dynamic Viscosity, kinematic viscosity, surface tension, capillary phenomenon.

FLUID MECHANICS

CHAPTER-1

PROPERIES OF FLUID

1.1 FLUID

Definition: A fluid is a substance which is capable of flowing or a substance which deforms continuously when subjected to external shearing force.

Characteristics:

- It has no definite shape of its own but will take the shape of the container in which it is stored.
- A small amount of shear force will cause a deformation.

Classification

Liquid : It is a fluid which possesses a definite volume and assumed as incompressible.

Gas : It possesses no definite volume and is compressible.

Fluids (Gas & Liquid) are broadly classified in to two types

- Ideal Fluid
- Read Fluid

Ideal Fluids: An ideal fluid is one which has no viscosity and surface tension and is incompressible. Actually no ideal fluid exists.

Real Fluid : A real fluid is one which has viscosity, surface tension and compressibility in addition to the density.

Fluid Mechanics :

Fluid Mechanics may be defined as that branch of Engineering science which deals with the behaviour of fluid under the conditions of rest and motion.

It may be divided in to three catagories.

- Fluid statics
- Fluid Kinematics
- Fluid Dynamics

Fluid Statics : The study of fluid at rest is called fluid statics.

- The study of incompressible fluids under static conditions is called Hydrostatics.
- The study of compressible fluids (gases) under static conditions is called Aerostatics

Fluid Kinematics : The study of fluids in motion where pressure forces are not considered is called fluid kinematics.

Fluid Dynamics : The study of fluids in motion where pressure forces are considered is called fluid dynamics

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Description of fluid properties like Density, specific weight, specific gravity, specific volume and solve simple problems

Density

- Density or Mass density of a fluid is defined as the ratio of the mass of a fluid to its volume
- It is denoted by ρ.
- The density of liquids are considered as constant while that of gases changes with pressure and temperature variations.

Mathematically

$$\rho = \frac{\text{Mass}}{\text{Volume}}$$

unit = kg/m^3

ρ_{water} = 1000 kg/m^3 or 1 gm/cm^3

Specific weight or weight Density (w)

- Specific weight of a fluid is defined as the ratio between the weight of a fluid to its volume
- Weight per unit volume of a fluid is also called weight density or specific weight.
- It is denoted by 'w'.

Mathematically $w = \frac{\text{weight of fluid}}{\text{Volume of fluid}}$

$= \rho g$ unit = N/m^3 (SI sys)

$\therefore$ Specific weight of water is 9810 N/m^3

Specific Volume

Specific Volume of a fluid is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid

Mathematically

$$\text{Specific volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}}$$

$$= \frac{1}{\frac{\text{Mass of fluid}}{\text{volume of fluid}}}$$

$$= \frac{1}{\rho}$$

Unit = m^3/kg (Reciprocal of density)

It is commonly applied to gases.

Specific gravity (S)

Specific gravity is defined as the ratio of the specific weight or density of a fluid to the specific weight or density of a standard fluid.

For liquids, the standard fluid is water

For gases, the standard fluid is Air

It is denoted by S

Mathematically

$$S(\text{for liquids}) = \frac{\text{Specific weight or density of a liquid}}{\text{specific weight or density of a standard liquid (water)}}.$$

$$S(\text{for gases}) = \frac{\text{Specific weight or density of a gas}}{\text{Specific weight or density of air}}$$

$$w_l = w_w \times S$$

$$\rho_l = \rho_w \times S \qquad \text{(It is a unitless quantity)}$$

Simple problems

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Q1 Calculate the specific weight, density and specific gravity of one litre of a liquid which weighs 7N.

Solution

Given datas

Volume = 1 lt $= 10^{-3}\ m^3$

Weight = 7 N

$$\text{Specific weight} = \frac{\text{Weight}}{\text{Volume}} = \frac{7\,N}{10^{-3}\,m^3} = 7000\ N/m^3$$

$$\text{Density} = \frac{\text{Specific weight}}{g} = \frac{w}{g} = \frac{7000}{9.81}\ kg/m^3$$

$$= 713.5\ kg/m^3$$

$$\text{Specific gravity} = \frac{\text{Density of liquid}}{\text{Density of water}}$$

$$= \frac{713.5}{1000} = 0.7135$$

Q.2 One litree of crude oil weighs 9.6 N. calculate its specific weight; density and specific gravity.

Solution

Volume of crude oil = 1 lt

$= 10^{-3}\ m^3$

Weight of the oil $= W = 9.6\ N$

$$\text{Specific weight} = w_l = \frac{W}{V} = \frac{9.6}{10^{-3}} = 9600\ N/m^3$$

$$\text{density} = \rho = \frac{w}{g} = \frac{9600}{9.81} = 978.6\ kg/m^3$$

$$\text{Specific gravity of oil} = s = \frac{w_l}{w_w}$$

$$= \frac{9600}{1000 \times 9.81} = 0.978$$

Q3 Calculate the specific weight, specific mass, speci[fic] volume and specific gravity of a liquid having a volume of $6\ m^3$ and weight of 44 kN.

Solution

Given

Volume of liquid = $V = 6\ m^3$

Weight of liquid = 44 kN = 44,000 N

Specific weight of the liquid

$$w_l = \frac{W}{V} = \frac{44{,}000}{6} = 7333\ N/m^3$$

Specific mass or density

$$\rho_l = \frac{w_l}{g} = \frac{7333}{9.81} = 747.5\ kg/m^3$$

Specific volume of the liquid

$$= \frac{1}{\rho} = \frac{1}{747.5} = 0.00134\ m^3/kg$$

Specific gravity of the liquid

$$S = \frac{\rho_l}{\rho_w} = \frac{747.5}{1000} = 0.7475$$

Ans

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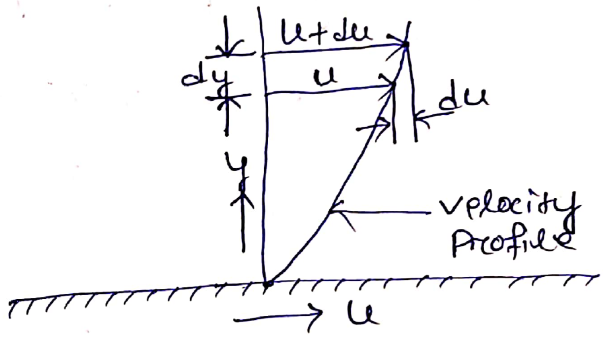
Definitions and units of Dynamic viscosity, kinematic viscosity, Surface tension, capillary Phenomenon

Viscosity

Viscosity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of the fluid.

Let us consider two layers of fluid at a distance dy apart, move one over the other at different velocities u and u+du



Velocity variation near a solid boundary

The viscosity with the relative velocity between the fluid layers causes a shear stress acting between the fluid layers.

The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer

The shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by τ.

Mathematically

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

where μ = Co-efficient of dynamic viscosity or constant of proportionality or viscosity

$\frac{du}{dy}$ = reate of sheare streain
or
velocity greadient

$\Rightarrow \mu = \frac{\tau}{du/dy}$

If $\frac{du}{dy} = 1$ then $\boxed{\mu = \tau}$

Viscosity is also defined as the sheare stress reequired to preoduce unit reate of sheare stream

unit of viscosity

In S.I system — Ns/m^2

CGS system — $Dyne.s/cm^2$

MKS system — $Kgf\ s/m^2$

1 $Dyne.sec/cm^2$ = 1 Poise

1 $N.s/m^2$ = 10 poise

1 centipoise = $\frac{1}{100}$ poise

Note: Viscosity of watere at 20°c is 0.01 poise or 1.0 centipoise.

Kinematic viscosity

It is defined as the reatio between the dynamic viscosity and density of fluid.

It is denoted by ν (nu).

Mathematically: $\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho}$

unit = m^2/sec (S.I) cm^2/sec (C.G.S)

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cm^2/sec is also known as stoke

$$1 \text{ stoke} = 1\ cm^2/sec$$
$$= \left(\frac{1}{100} m\right)^2 / sec = 10^{-4}\ m^2/sec$$

$$\text{centistoke} = \frac{1}{100} \text{ stoke.}$$

NOTE

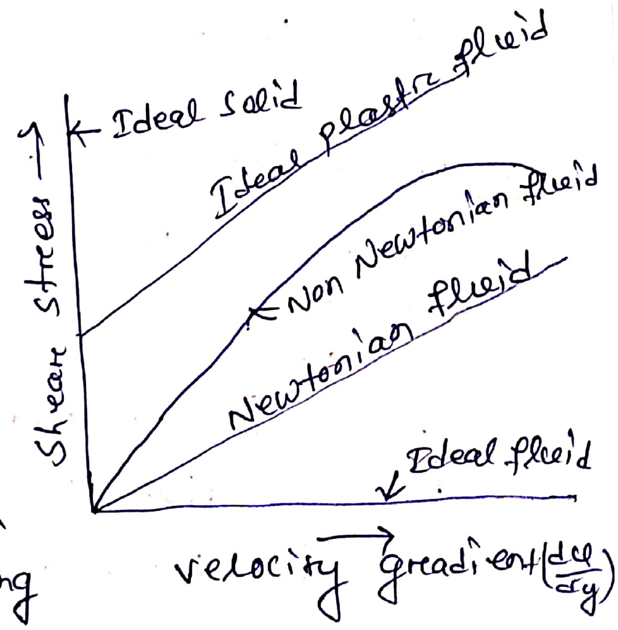
Newton's Law of Viscosity

It states that the sheare stress (τ) on a fluid element layere is directly proportional to the rate of sheare strain. The constant of proportionality is called the co-efficient of viscosity.

$$\tau = \mu \frac{du}{dy}$$

Types of fluids

- Ideal fluid
- Real fluid
- Newtonian fluid
- Non - Newtonian fluid
- Ideal plastic fluid.



Ideal Fluid: A fluid which is incompressible and is having no viscosity is known as an ideal fluid. It is only an imaginary fluid.

Real Fluid

A fluid which possesses viscosity is known as real fluid. All the fluids are real fluid in actual practice.

Newtonian Fluid

A real fluid in which the shear stress is directly proportional to the rate of shear strain is known as a Newtonian fluid.

Non Newtonian Fluid

A real fluid in which shear stress is not proportional to the rate of shear strain is known as Non-Newtonial fluid.

Ideal Plastic Fluid:

A fluid in which shear stress is more than the yield value and shear stress is proportional to the rate of shear strain is known as ideal plastic fluid.

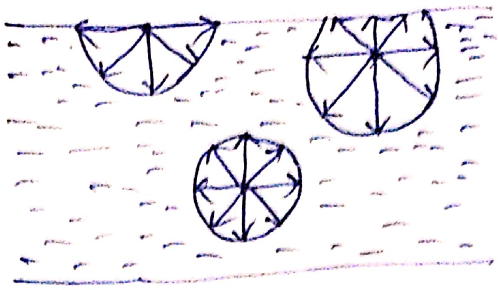
Surface Tension

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension.

It is denoted by σ.

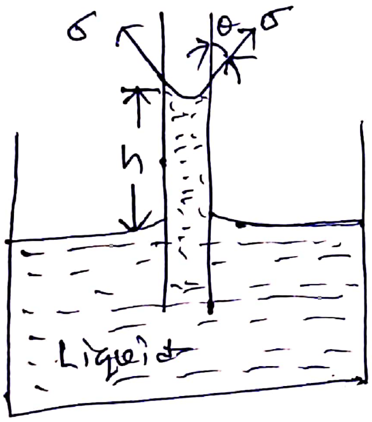
It is represented as force per unit length or of energy per unit area.

Unit = kgf/m (M.K.S) or N/m (S.I)

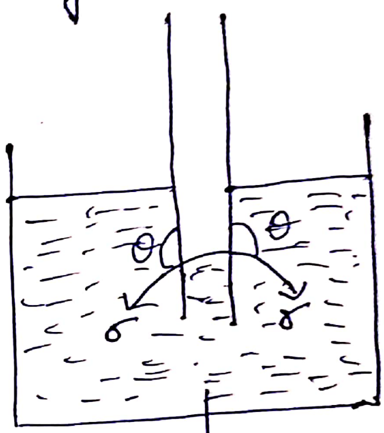


Capillarity

Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid.



Capillary rise



capillary fall.

The rise of liquid surface in the tube is known as capillary rise; while the fall of liquid surface is known as capillary depression.

It is expressed in terms of cm or mm of liquid.

It's ~~express~~ value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.

CHAPTER - 2

FLUID PRESSURE AND ITS MEASUREMENTS

2.1 Definitions and units of fluid pressure, Pressure intensity and pressure head.

2.2 Statement of Pascal's Law.

2.3 Concept of atmospheric pressure, gauge pressure, vacuum pressure and absolute pressure.

2.4 Pressure Measuring instruments Manometers (simple and Differential)

2.4.1 Bourdon tube Pressure gauge (simple Numericals)

2.5 Solve simple problems on Manometer

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CHAPTER-2

FLUID PRESSURE AND ITS MEASUREMENTS

2.1 Definition and units of fluid pressure, pressure Intensity and pressure head.

Pressure of a Fluid

When a fluid is contained in a vessel, it exerts force at all points on the sides & bottoms of the container. The force exerted per unit area is called pressure.

If p = Pressure or intensity of pressure

F = Force

A = Area on which force acts

$$p = F/A$$

Unit of pressure – Kgf/m^2 in MKS

N/m^2 in S.I

$Dyne/cm^2$ in CGS

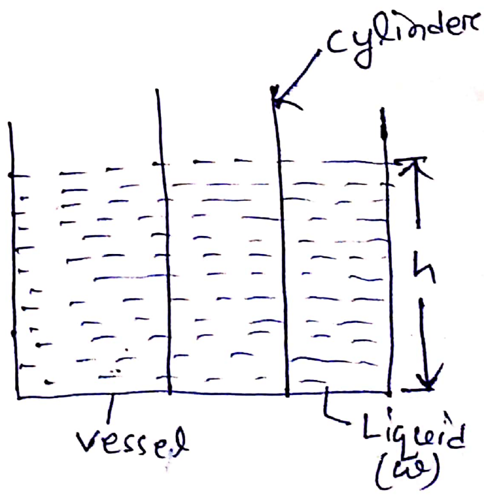
Pressure is always expressed in pascal.

$$\boxed{1 \text{ Pascal} = 1\ N/m^2}$$

Pressure Head of a liquid

A liquid is subjected to pressure due to its own weight, this pressure increases as the depth of the liquid increases.

Let a bottomless cylinder stands in the liquid



Let w = Specific weight of the liquid
h = height of the liquid in the cylinder
A = Area of the cylinder

$$P = F/A$$

$$= \frac{\text{Weight of the liquid in the cylinder}}{\text{Area of the cylinder}}$$

$$= \frac{w \times V}{A}$$

$$= \frac{w \times A \times h}{A} = wh$$

$$= \rho g h \quad (w = \rho g)$$

$$\boxed{P = \rho g h}$$

So intensity of pressure or pressure at any point in a liquid is proportional to its depth.

1.2 Statement of Pascal's Law

It states that " The intensity of pressure at any point in a fluid at rest is the same in all directions".

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Calculate the pressure due to a column of 0.3M of (a) water, (b) an oil of sp. gr. 0.8 and (c) Mercury of sp. gr 13.6. Take density of water $\rho = 1000\ kg/m^3$

Solution

Height of liquid column = 3 m

(a) For water

$$\text{pressure} = p = \rho g h$$
$$= 1000 \times 9.81 \times 3$$
$$= 29430\ N/m^2$$

(b) For an oil of sp. gravity 0.8

$$p = \rho_o g h$$
$$= (\rho_w \times S_o)\, g h$$
$$= 1000 \times 0.8 \times 9.81 \times 3$$
$$= 23544\ N/m^2$$

(c) For Mercury

$$p = \rho_{Hg}\, g h$$
$$= 13600 \times 9.81 \times 3$$
$$= 400248\ N/m^2$$

Q2 The pressure intensity at a point in a fluid is given $3.924\ N/cm^2$. Find the corresponding height of fluid when the fluid is (a) water, (b) oil of sp. gr 0.9.

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Solution

Pressure intensity $P = 3.924\ N/cm^2$
$= 3.924 \times 10^4\ N/m^2$

The corresponding height h, of the fluid is given by $\boxed{h = \frac{P}{\rho g}}$

(a) For water

$$h = \frac{P}{\rho_w g} = \frac{3.924 \times 10^4}{1000 \times 9.81} = 4\ m \text{ of water}$$

(b) For oil of sp. gre 0.9

$S_o = 0.9$

$\rho_o = \rho_w \times S_o = 1000 \times 0.9 = 900$

$$h = \frac{P}{\rho_o g} = \frac{3.924 \times 10^4}{900 \times 9.81}$$
$= 4.44\ m$ of oil.

Q3 An oil of sp. gravity 0.9 is contained in a vessel. At a point the height of oil is 40 m. Find the corresponding height of water at that point.

Solution

$S_o = 0.9$ $\quad \rho_o = S_o \times \rho_w$
$= 0.9 \times 1000 = 900\ kg/m^3$

$h = 40\ m$

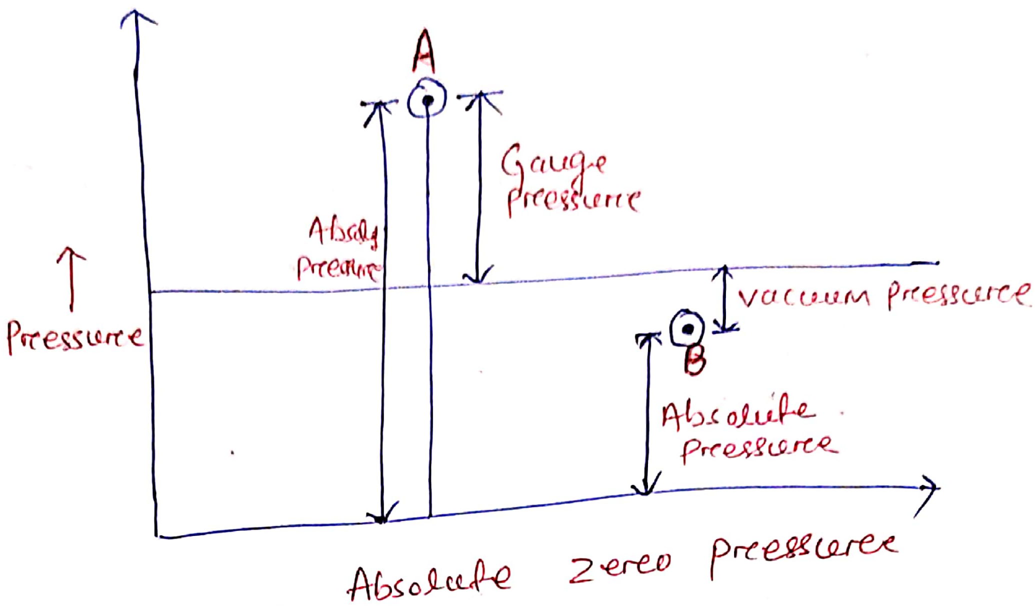
$P = \rho_o g h = 900 \times 9.81 \times 40$
$= 353160\ N/m^2$

In case of water
$P = \rho_w g h$

$$h = \frac{P}{\rho_w g} = \frac{353160}{1000 \times 9.81} = 36\ m \text{ of water}$$

Ans

Concept of Atmospheric pressure, Gauge Pressure Vacuum pressure and absolute pressure.



Atmospheric Pressure

The atmospheric air exerts a normal pressure upon all surfaces with which it is in contact and this pressure is known as atmospheric pressure.

Absolute pressure:

It is defined as the pressure which is measured with reference to absolute vacuum pressure or absolute zero pressure.

Gauge pressure

It is defined as the pressure which is measured with the help of a pressure measuring instrument in which atmospheric pressure is taken as datum. Atmospheric pressure on scale is marked as zero.

Vacuum pressure

It is defined as the pressure below atmospheric pressure.

Absolute pressure = Atmospheric pressure + gauge pressure

$$\boxed{P_{abs} = P_{atm} + P_{gauge}}$$

Vaccum pressure = Atmospheric Pressure − Absolute pressure

$$\boxed{P_{vaccum} = P_{atm} - P_{abs}}$$

2.4 Pressure Measuring Instruments

The pressure of a fluid is measured by the following two devices.

- Manometers
- Mechanical Gauges

Manometers

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same ore another column of the fluid. They are classified as

(a) simple Manometer

(b) Differential Manometer

Mechanical Gauges

Mechanical gauges are defined as the devices used for measuring the Pressure by balancing the

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The fluid column by the spring or dead weight.
The commonly used Mechanical pressure gauges are

(a) Diaphragm Pressure gauge
(b) Bourdon tube pressure gauge
(c) Dead-weight pressure gauge
(d) Bellows pressure gauge.

MANOMETERS

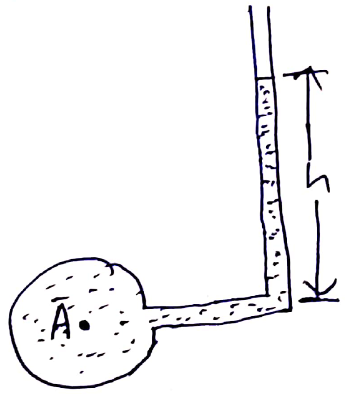
Simple Manometer

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere.

Common type of simple manometers are

1. piezometer
2. U-tube manometer
3. single column manometer

Piezometer



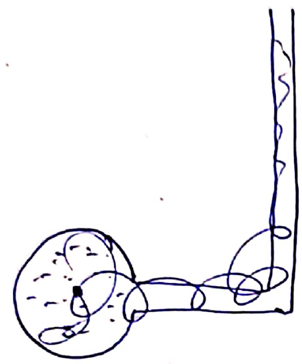
- It is the simplest form of manometer used.
- One end is connected to the point where the pressure is to be measured and the other end is open to the atmosphere.
- The rise of liquid gives the pressure head of that point.

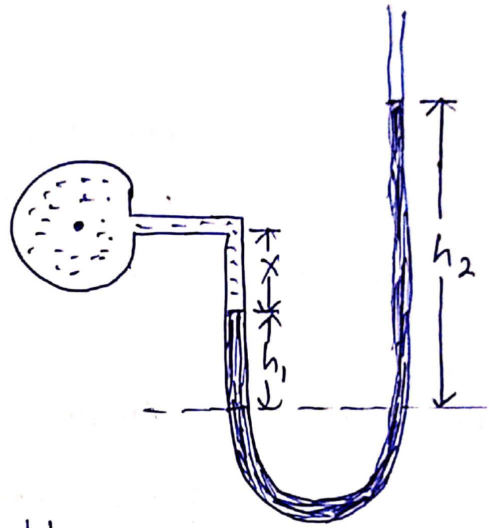
If at a point A, the height of liquid is h, then
Pressure $P = \rho g h$

U-tube Manometer

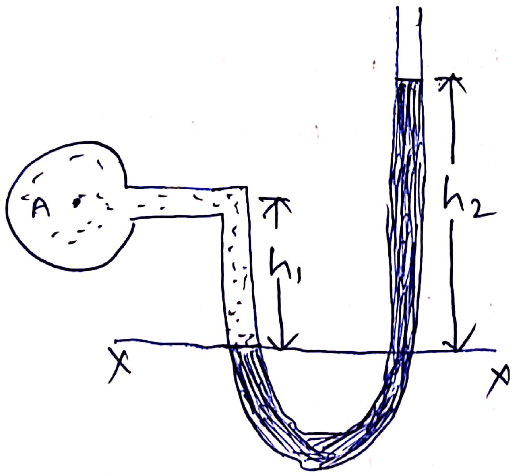
- It consists of glass tube bent in U-shape, one end of the tube is connected to a point where the pressure is to be measured and other end remains open to the atmosphere.

- The tube generally contains mercury or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.





For gauge pressure



For gauge Pressure.

Lets A is the point where the Pressure is to be measured

The datum line is X - X

Let h_1 = Height of light liquid above the datum line

h_2 = Height of heavy liquid above the datum line

S_1 = Sp. gravity of light liquid

ρ_1 = density of light liquid = $S_1 \times 1000$

S_2 = Sp. gravity of heavy liquid

ρ_2 = Density of heavy liquid = $S_2 \times 1000$

As the pressure is the same for the horizontal surface. The pressure above the horizontal datum line X-X in the left column and in the right column of U-tube Manometer should be same.

Pressure above X-X in the left column

$$= P_A + \rho_1 g h_1$$

Pressure above X-X in the right column

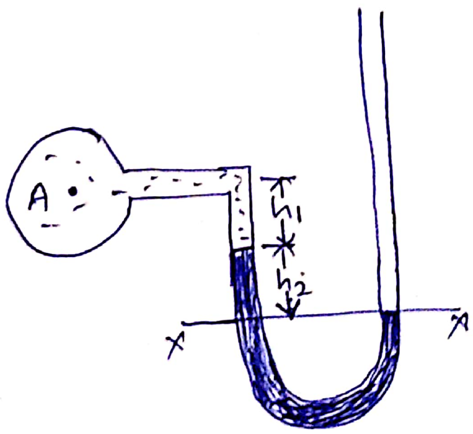
$$= \rho_2 g h_2$$

equating both

$$P_A + \rho_1 g h_1 = \rho_2 g h_2$$

$$P_A = \rho_2 g h_2 - \rho_1 g h_1$$

For Vacuum Pressure



Pressure above the datum line in the left column and right column is same.

Pressure above X-X in the left column

$$= P_A + \rho_1 g h_1 + \rho_2 g h_2$$

Pressure above X-X in the right column is zero

$$\Rightarrow P_A + \rho_1 g h_1 + \rho_2 g h_2 = 0$$

$$\Rightarrow P_A = -(\rho_1 g h_1 + \rho_2 g h_2)$$

Single Column Manometere

- It is a modified forem of a U-tube Manometere in which a reeserevoire, having a larege creoss-sectional areea (about 100 times) as compareed to the areea of the tube is connected to one of the limbs of the Manometere.

- Due to larege creoss-sectional areea of the reeserevoire, fore any variation in Preessuree, the change in the liquid level in the reesorevoire will be verey small which may be neglected and hence the preessuree is given by the height of liquid in the othere limb.

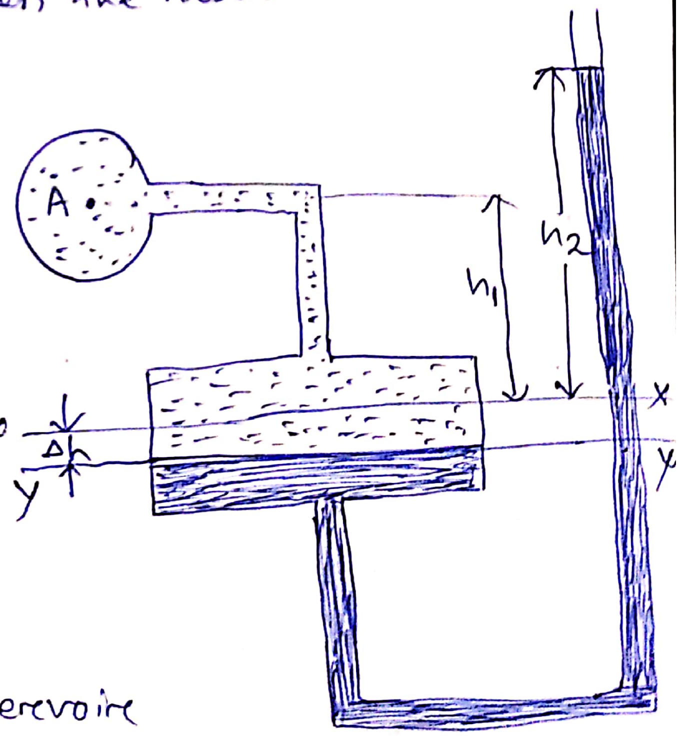
There are two types of single column Manometere

1. Verertical single column Manometere
2. Inclined single column Manometere.

Verefical single column Manometere

X-X is the datum line when the reeserevoire is not Connected to the pipe.

Y-Y is the datum line in the reeserevoire when it is Connected to the pipe.



Let Δh = Fall of heavy liquid in reeserevoire.

h_2 = Rise of heavy liquid in reight limb.

h_1 = Height of centree of Pipe above X-X

P_A = Preessuree at A

A = C.S areea of the reeserevoire

a = C.S areea of the reight limb

S_1 = sp. greavity of liquid in pipe

S_2 = sp. gravity of liquid in the reeserevoire.

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a h_2}{A}$$

The pressure in the right limb above $y-y$

$$= \rho_2 g (h_2 + \Delta h)$$

Pressure in the left limb above $y-y$

$$= P_A + \rho_1 g (h_1 + \Delta h)$$

Equating both the pressure

$$\Rightarrow \rho_2 g (h_2 + \Delta h) = P_A + \rho_1 g (h_1 + \Delta h)$$

$$\Rightarrow P_A = \rho_2 g h_2 + \rho_2 g \Delta h - \rho_1 g h_1 - \rho_1 g \Delta h$$

$$\Rightarrow \rho_2 g h_2 - \rho_1 g h_1 - \Delta h (\rho_2 g - \rho_1 g)$$

$$\Delta h = \frac{a \times h_2}{A}$$

$\frac{a}{A}$ becomes very small and can be neglected

$$\Rightarrow P_A = \rho_2 g h_2 - \rho_1 g h_1$$

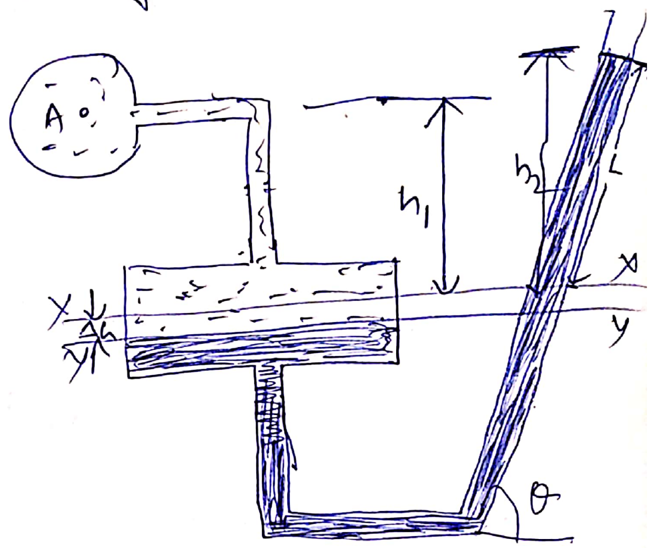
Inclined single column Manometere

L = Length of heavy liqueid moved in reight limb from x - x

θ = Angle of inclination of reight limb

h_2 = Veretical reise of heavy liquid in right limb from x-x

= $L \sin\theta$



$$P_A = \rho_2 g h_2 - \rho_1 g h_1$$

$$= \rho_2 g L \sin\theta - \rho_1 g h_1$$

DIFFERENTIAL MANOMETERS

- Differeential Manometeres aree the devices used fore Measureing the differeence of preessurees between two points in a pipe ore in two differeent pipes.

- A differeential manometere consists of a U-tube, containing a heavy liqueid, whose two ends aree connected to the points, whose differeence of preessuree is to be measureed.

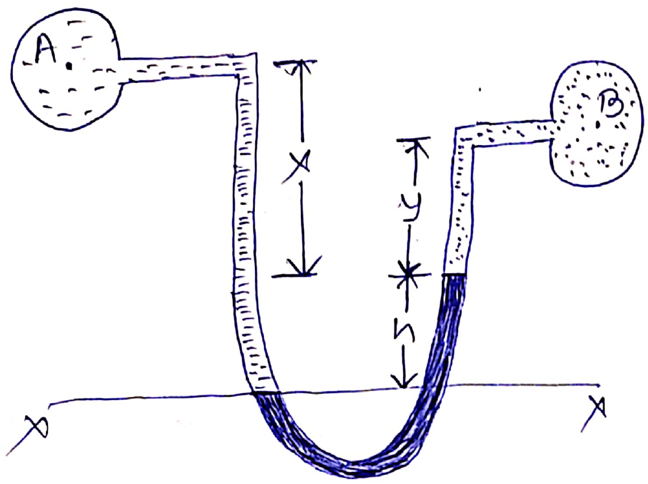
Most common types of differeential Manometeres aree

- U-tube differeential Manometere
- Inveretted U-tube differeential Manometere

U-tube Differential Manometer

Case-I

Two points are at different level and also contains liquids of different sp. gravity.



The two points A, B are connected to the U-tube Manometer. Let the pressure at A & B are P_A & P_B

Let h = Difference of mercury level in the U-tube

y = Distance of the centre of B from the mercury level in the right limb.

x = Distance of the centre A, from the mercury level in the right limb.

ρ_1 - density of liquid at A

ρ_2 = density of liquid at B

ρ_g = density of heavy liquid in the Manometer

Pressure above datum X-X in the left limb.

$$P_A + \rho_1 g(x+h)$$

Pressure above datum X-X in the right limb

$$P_B + \rho_2 g y + \rho_g g h$$

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Equating both the pressures

$$P_A + \rho_1 g(x+h) = P_B + \rho_2 g y + \rho_g g h$$

$$\Rightarrow P_A - P_B = \rho_2 g y + \rho_g g h - \rho_1 g x - \rho_1 g h$$

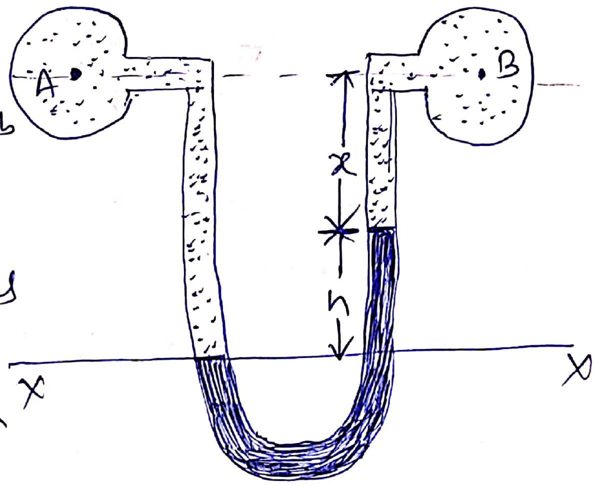
$$= \rho_2 g y - \rho_1 g x + gh(\rho_g - \rho_1)$$

Case-II

The two points A & B are at the same level and contains the same liquid of density ρ_1.

Pressure above X-X in left limb.

$$P_A + \rho_1 g(x+h)$$



Pressure above X-X in right limb

$$P_B + \rho_1 g x + \rho_g g h$$

equating both the pressures

$$\Rightarrow P_A + \rho_1 g(x+h) = P_B + \rho_1 g x + \rho_g g h$$

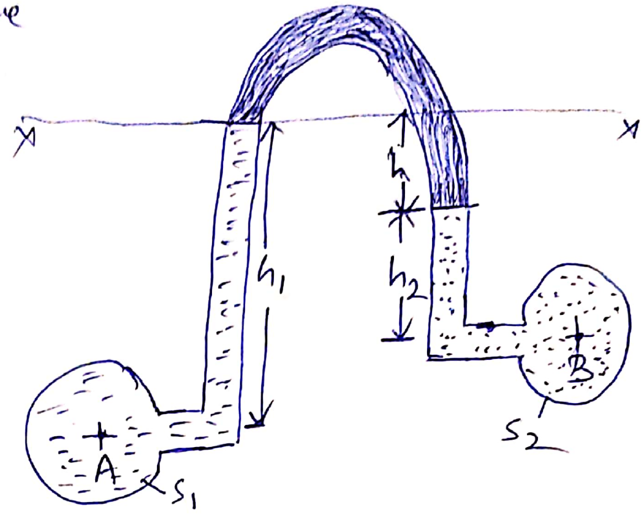
$$\Rightarrow P_A + \cancel{\rho_1 g x} + \rho_1 g h = P_B + \cancel{\rho_1 g x} + \rho_g g h$$

$$\Rightarrow P_A - P_B = \rho_g g h - \rho_1 g h$$

$$= gh(\rho_g - \rho_1)$$

Inverted U-tube Differential Manometer.

- It consists of an inverted U-tube containing a light liquid
- The two ends of the tube are connected to the points whose difference of pressure is to be measured.
- It is used for measuring difference of low pressures.



Let the pressure at A is more than pressure at B

Let h_1 = Height of liquid A in left limb below the datum line X-X

h_2 = Height of liquid B in right limb

h = difference of light liquid

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_s = Density of light liquid

P_A = Pressure at A

P_B = Pressure at B

Pressure in the right limb

$$P_B - \rho_2 g h_2 - \rho_s g h$$

Pressure in the left limb

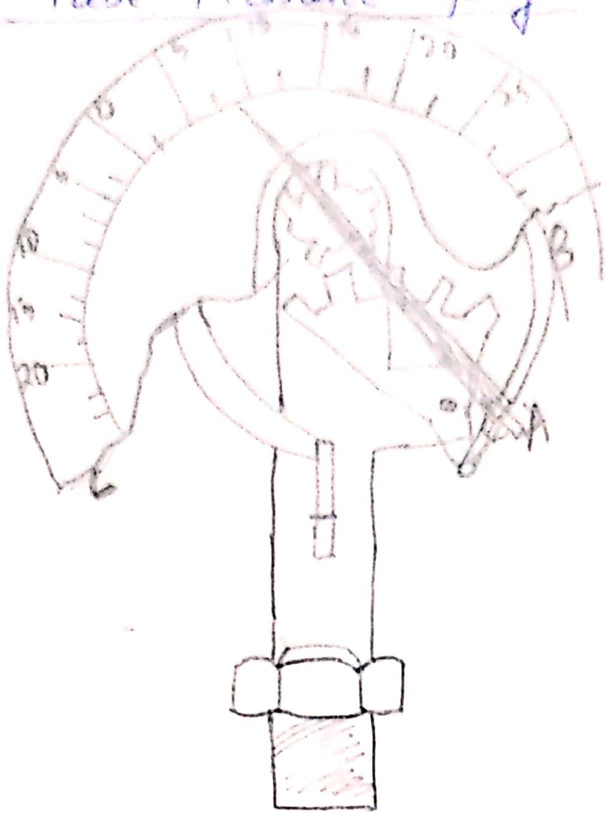
$$P_A - \rho_1 g h_1$$

Equating both the pressures

$$P_B - \rho_2 g h_2 - \rho_s g h = P_A - \rho_1 g h_1$$

$$\Rightarrow P_A - P_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_s g h$$

Bourdon's Tube Pressure Gauge



- The pressure above or below atmospheric pressure may be easily measured with the help of bouredon tube pressure gauge.

- It consists of an elliptical tube ABC bent into an arc of a circle. This bent up tube is called bouredon tube.

- when the gauge tube is connected to the fluid, the fluid under pressure flows in to the tube the bouredon tube as a result of the increase pressure tends to straighten itself.

- Since the tube is encased in a circular cover therefore it tends to become circular instead of straight.

- The elastic deformation of the bouredon tube rotates the pointer.

- The pointer moves over a calibrated scale which directly gives the pressure,

HYDROSTATICS CHAPTER-3

3.1 Definition of hydreostatic preessurce

3.2 Total preessuree and centree of preessurce on immeresed bodies (Horeizontal and verefical Bodies)

3.3 Solve simple preoblems

3.4 Archimedes Preinciple, concept of bueoyancy, Meta centere and Meta centreic height. (Definition only)

3.5 Concept of floatation

Senior Lecturer
Mechani-

Introduction

- Hydrostatic Means fluids (Liquids and gases) at rest.
- There will be no relative Motion between adjacent fluid layers.
- The velocity gradient will be zero as well as shear stress.
- The forces Acting on the fluid particles will be due to
  1. Pressure of fluid normal to surface
  2. Gravity (self weight of fluid particles)

Total Pressure and Centre of pressure on Immersed. Bodies

Total pressure

- It is defined as the force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces.
- This force always acts normal to the surface.

Centre of pressure

It is defined as the point of application of the total pressure on the surface.

There are four cases of submerged surfaces.

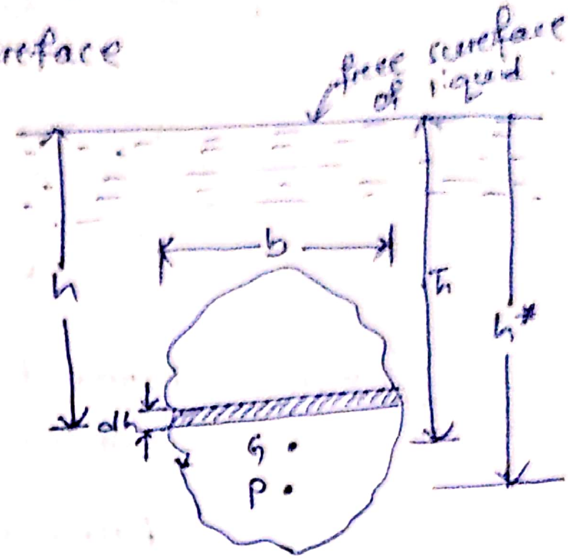
1. Vertical plane surface
2. Horizontal plane surface
3. Inclined plane surface
4. Curved surface

Senior Lecturer
Mech

Vertical plane surface submerged in liquid.

Consider a plane vertical surface of arbitrary shape immersed in a liquid.

Let A = Total area of the surface



$\bar{h}$ = Distance of C.G. of the area from free surface of liquid

G = Centre of gravity of plane surface

P = Centre of pressure

h^* = Distance of centre of pressure from free surface of liquid.

Total pressure

The total pressure on the surface may be determined by dividing the entire surface into a number of small parallel strips. The force on entire surface is then calculated by integrating the force on small strip.

Consider a strip of thickness dh & width b at a depth of h from free surface of liquid.

Pressure intensity on the strip

$$p = \rho g h$$

Area of the strip = $b \times dh = dA$

The pressure force on strip $dF = p\,dA$

$= \rho g h \times b \times dh$

Then the total pressure force on the whole surface

$$F = \int dF = \int \rho g h \times b \times dh$$

$$= \rho g \int b \times h \times dh$$

$$= \rho g \int h \times dA$$

$\int h \times dA$ = Moment of surface area about free surface of liquid.

= Area of the surface × distance of CG from free surface of liquid.

$$= A \times \bar{h}$$

$$F = \rho g A \bar{h}$$

Centre of pressure (h^*)

Centre of Pressure is calculated by using the 'Principle of Moments', which states that the moment of resultant force about an axis is equal to the sum of moments of the components about the same axis.

The resultant force F is acting at P, at a distance h^* from the free surface of liquid.

Hence moment of force F. about the free surface of liquid $= F \times h^*$

But Moment force dF = acting on a strip about the free surface of liquid

$= dF \times h$.

Sum of moments of all such forces about free surface of liquid.

$$= \int \rho g h \times b \times dh \times h$$

$$= \rho g \int b \times h \times dh \times h$$

$$= \rho g \int b h^2 dh$$

$$= \rho g \int h^2 dA$$

$\int h^2 dA$ = Moment of inertia of the surface area about free surface of liquid.

$= I_0$

Sum of moments about free surface

$$= \rho g I_0$$

i.e

$$F \times h^* = \rho g I_0$$

$$\Rightarrow \rho g A \bar{h} \times h^* = \rho g I_0$$

$$\Rightarrow h^* = \frac{\cancel{\rho g} I_0}{\cancel{\rho g} A \bar{h}} = \frac{I_0}{A \bar{h}}$$

By parallel axis theorem

$$I_0 = I_G + A \times \bar{h}^2$$

I_G = Moment of inertia of the surface about an axis passing through the C.G of the area and parallel to free surface of liquid.

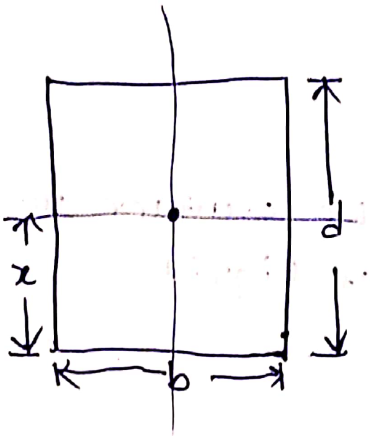
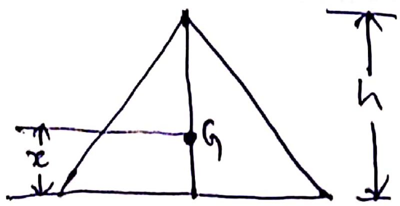
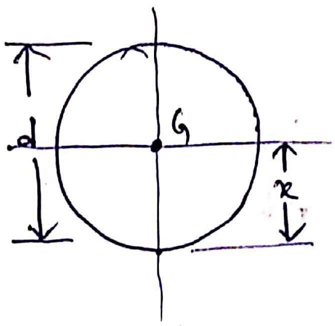
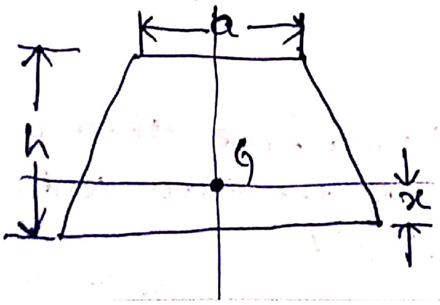
Senior Lecturer

Substitute the value of I_o

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}}$$

$$\boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$

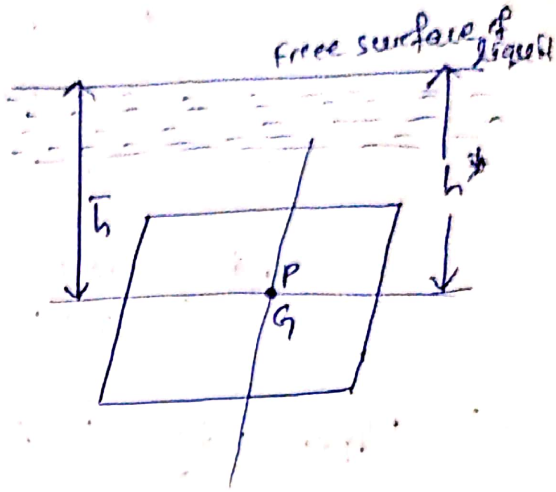
Common surfaces and their centre of gravity, Moment of Inertia about an axis I_o.

Surfaces	C.G. from base	Area	M.I about C.G (I_G)	I_o
	$x = d/2$	bd	$\frac{bd^3}{12}$	$\frac{bd^3}{3}$
	$x = \frac{h}{3}$	$\frac{bh}{2}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
	$x = \frac{d}{2}$	$\frac{\pi}{4}d^2$	$\frac{\pi d^4}{64}$	
	$x = \frac{(2a+b)h}{(a+b)\,3}$	$\frac{(a+b)h}{2}$	$\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^3$	

Horizontal Plane surface submereged in liquid.

Consider a plane horizontal surface immeresed in a static fluid.

As every point of the surface is at the same depth from the free surface of the liquid the pressure intensity will be equal on the entire surface.



$$P = \rho g \bar{h}$$

$$A = \text{total area}$$

$$F = P \times A = \rho g \bar{h} A$$

Simple Numericals

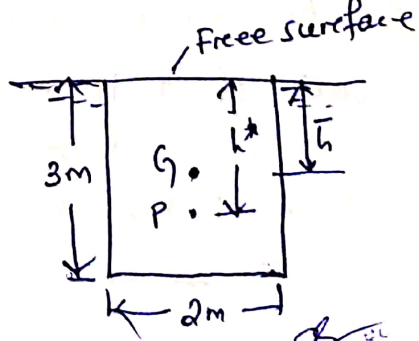
Q.1 A rectangulare plane surface is 2m wide and 3m deep. It lies in veretical plane in watere. Deteremine the total preessure and position of centre of pressure on the plane surface when its uppere edge is horeizontal and (a) Coincides with watere surface (b) 2.5m below the free watere surface

Solution

Width of plane = $b = 2m$

Depth of plane = $d = 3m$

(a) Uppere edge coincides with watere surface



$$F = \rho g A \bar{h}$$

$$\rho = 1000 \ kg/m^3$$

$$A = b \times d = 2 \times 3 = 6m^2$$

$$\bar{h} = \frac{1}{2}(3) = 1.5 \ m$$

$$F = 1000 \times 9.81 \times 6 \times 1.5$$
$$= 88290 \text{ N}$$

Depth of centre of pressure

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$I_G = \frac{bd^3}{12} = \frac{2\times3^3}{12} = 4.5 \text{ m}^4$$

$$h^* = \frac{4.5}{6\times1.5} + 1.5 = 0.5 + 1.5 = 2.0 \text{ m}$$

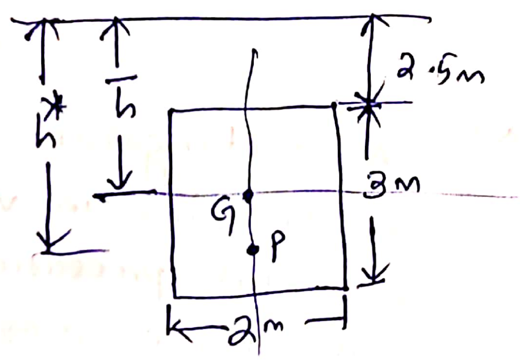
(b) Upper edge is 2.5 m below water surface

$$F = \rho g A \bar{h}$$

where $\bar{h}$ = Distance of C.G from free surface.

$\bar{h} = 2.5 + 1.5 = 4 \text{ m}$

$A = 2\times3 = 6 \text{ m}^2$



$$F = 1000 \times 9.81 \times 6 \times 4$$
$$= 235440 \text{ N}$$

centre of pressure

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$= \frac{4.5}{6\times4} + 4 = 4.1875 \text{ m}$$

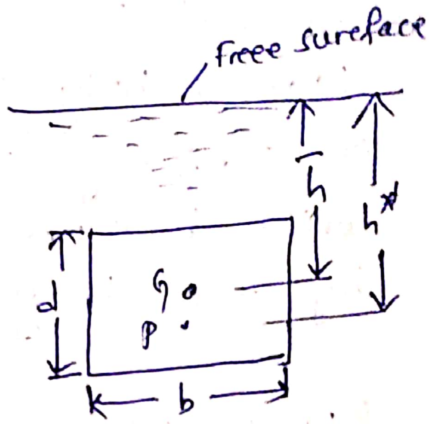
Ans

Q2 A rectangular sluice gate is situated on the vertical wall of a lock. The vertical side of the sluice is 'd' metres in length and depth of centroid of the area is 'p' m below the water surface. Prove that the depth of pressure is equal to $\left(p + \frac{d^2}{12p}\right)$

Solution

Depth of veretical gate = d m
width of veretical gate = b m

Areea $A = b \times d \ m^2$



Depth of C.G freom freee sureface

$\bar{h} = p\,m$

Let h^* is depth of centree of preessuree

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$I_G = \frac{bd^3}{12}$$

$$h^* = \frac{\frac{bd^3}{12}}{d \times b \times p} + p$$

$$= \frac{d^2}{12p} + p$$

$$\boxed{h^* = p + \frac{d^2}{12p}}$$

Preoved.

Q3 Deteremine the total preessuree on a cireculare plate of diametere 1·5m which is placed veretically in watere in such a way that the centree of the plate is 3m below the free surface of watere. Find the position of centree of preessuree also.

Senior Tortoise

Archimedes Principle

When a body is immersed in a liquid either wholly or partially it is buoyed or lifted up by a force which is equal to the weight of fluid displaced by the body.

Buoyancy

Whenever a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

Centre of Buoyancy.

It is defined as the point through which the force of buoyancy is supposed to act. The force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body. Centre of buoyancy will be the centre of gravity of the fluid displaced.

Meta-Centre

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle.

The Meta centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body, when the body is given a small

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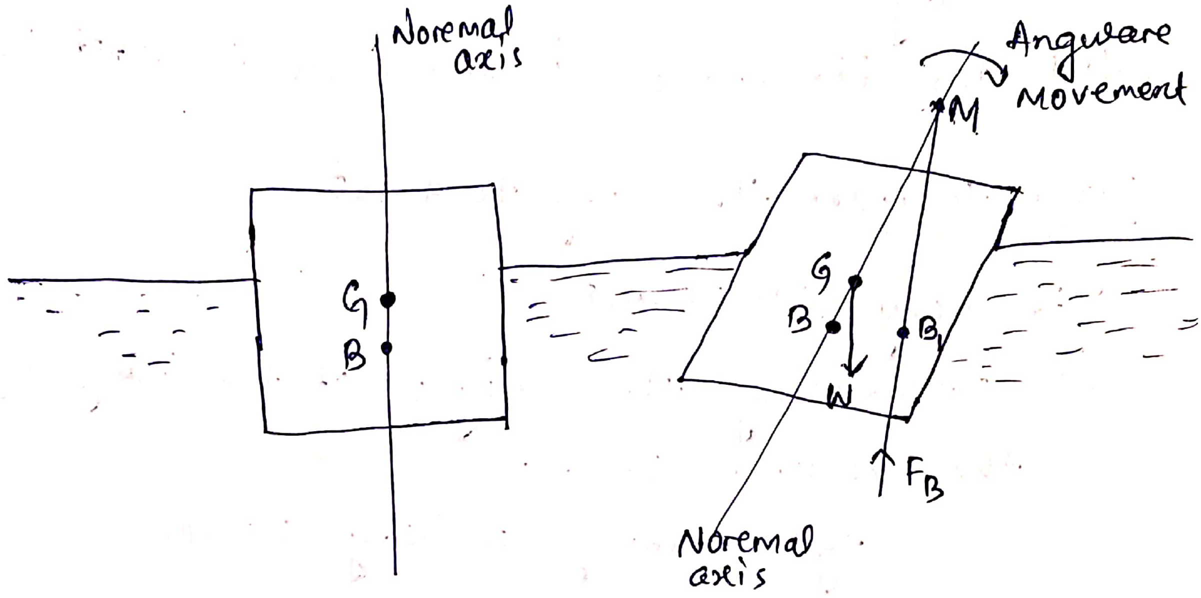
Mechani-

angulare displacement.



M = Meta centree

Metacentreic Height

The distance MG i.e the distance between the Meta centree of a floating body and the centre of gravity of the body is called Meta centreic height.

Concept of floatation

when a body immeresed in any fluid is experienced 2 forcees.

- Weight of the body acting veretically downwared
- Buoyancy foreee F_B acting veretically upward.

In case $W > F_B$ - The body will sink in the fluid

$W = F_B$ = The body will remain in equilibreium at any level.

$W < F_B$ = The body will move upwared in the fluid until the fluid displaced by its submerged part is equal to its weight(W). The body in this situation is called floating body and this phenomenon is known as floatation.

Ways to make the body to float

The body can float

(i) Decreasing the weight of the body by keeping the volume same.

eg: Making a body hollow.

(ii) Increasing the volume of the body by keeping the weight same.

eg: Attaching the life jacket to a person keeps the person floating.

Types of equilibrium of floating body.

- Stable equilibrium
- Unstable equilibrium
- Neutral equilibrium

① Stable Equilibrium

If a body floating in a liquid returns back to its original position when given a small angular displacement then the body is said to be in stable equilibrium.

② Unstable equilibrium

If a body floating in a liquid does not return back to its original position and heels farther away when given a small angular displacement, then the body is said to be in unstable equilibrium

③ Neutral equilibrium

If a body floating in a liquid occupies a new position and remains at rest in this new position when given a small angular displacement, then the body is said to be in neutral equilibrium.

Senior Lecturer

CHAPTER – 4

Kinematics of Flow

4.1 Types of fluid flow

4.2 Continuity equation (statement and proof for one dimensional flow)

4.3 Bernoulli's theorem (Statement and proof)

Applications and limitations of Bernoulli's theorem (Venturimeter, Pitot tube)

4.4 Solve simple problems

4.1 Types of fluid flow

The fluid flow is classified as

- Steady & Unsteady flow
- Uniform & Non uniform flow
- Laminar & turbulent flow
- Compressible & incompressible flow
- Rotational & irrotational flow
- One, two & three dimensional flow

Steady & Unsteady Flow

Steady flow is defined as that type of flow in which the fluid characteristic like velocity pressure, density etc at a point do not changes with time.

Thus for steady flow

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} = 0 \qquad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

$$\left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} = 0$$

where x_0, y_0, z_0 is a fixed point in fluid field.

Unsteady flow is defined as that type of flow in which the fluid characteristics like velocity pressure, density etc at a point changes with respect to time. Thus for unsteady flow

$$\left(\frac{\partial v}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \quad \left(\frac{\partial p}{\partial t}\right)_{x_0, y_0, z_0} \neq 0, \quad \left(\frac{\partial \rho}{\partial t}\right)_{x_0, y_0, z_0} \neq 0$$

Uniform & Non-Uniform Flow

Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space.

Mathematically

$$\left(\frac{\partial v}{\partial s}\right)_{t=constant} = 0.$$

∂v = change in velocity
∂s = Length of flow

Non-Uniform Flow is that type of flow in which the velocity at any given time changes with respect to space.

$$\left(\frac{\partial v}{\partial s}\right)_{t=constant} \neq 0$$

Laminar and Turbulent Flow

Laminar flow is defined as that type of flow in which the fluid particles move along well-defined paths or stream line and all stream lines are straight and parallel

Turbulent Flow is that type of flow in which the fluid, particles move in a zig-zag way.

For a pipe flow, the type of flow is determined by Reynold number

$$R_e = \frac{VD}{\nu}$$

where R_e = Reynold's Number
V = Mean velocity of flow
ν = Kinematic Viscosity

If $R_e < 2000$ flow is laminar.
If $4000 < R_e$ flow is turbulent
If $2000 < R_e < 4000$ flow is either laminar or turbulent

Compressible and Incompressible Flows

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Compressible flow is that type of flow in which the density of the fluid changes from point to point i.e density is constand fore the fluid.

$$\rho \neq \text{constant}$$

Incompressible flow is that type of flow in which the density is constant fore the fluid flow. Liquids are generally incompressible but gases are compressible.

$$\rho = \text{constant.}$$

Rotational and Irrotational Flows

Rotational flow is that type of flow in which the fluid particles while flowing along stream lines also rotate about their own axis.

Irrotational Flow is that type of flow in which the fluid particles while flowing along stream lines do not rotate about their own axis.

One, Two and Three-dimensional flow

One-dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and on space co-ordinate only.

$$u = f(x), \quad v = 0 \ \& \ w = 0$$

u, v, w are velocity components in x, y & z directions.

Two - dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates.

$$u = f_1(x, y), \quad v = f_2(x, y) \quad \text{and} \quad w = 0$$

Three - dimensional flow is that type of flow in which the velocity is a function of time and three space co-ordinates.

$$u = f_1(x, y, z), \quad v = f_2(x, y, z) \quad w = f_3(x, y, z)$$

Rate of Flow or Discharege (Q)

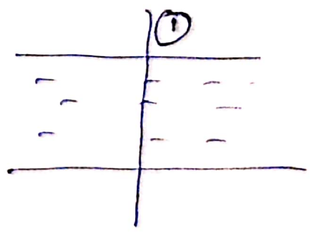
Rate of Flow Ore Discharege is defined as the quantity of a fluid flowing pere second through a section of a pipe ore a channel.

For incompressible fluid (Liquid)

Discharege(Q) is expressed as the volume of fluid flowing across the section pere second.

units of Q are m^3/s ore litres/s

Consider a liquid flowing through a pipe in which



A = C.S area of the pipe.

V = Average velocity of fluid

$$Q = A \times V$$

For Compressible Fluids (Gases)

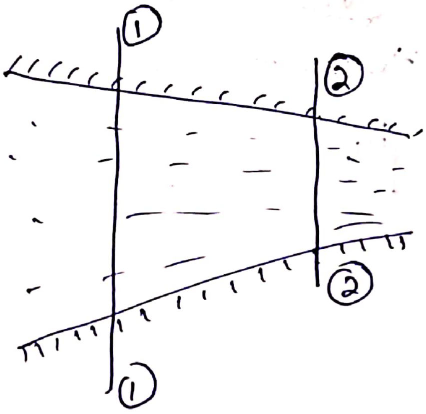
Discharege (Q) is expressed as weight of the fluid flowing across the section pere second.

units of Q are Kgf/s ore N/s

CONTINUITY EQUATION

Senior Lectures

The equation based on the principle of conservation of mass is called continuity Equation. Theres fore a fluid flowing through the pipe at all the cross section, the quantity of fluid per second is constant.



Let V_1 = Average velocity at cross section 1-1

ρ_1 = Density of section 1-1

A_1 = Area of pipe at Section 1-1

Similarly V_2, ρ_2, A_2 are the corresponding values at section 2-2.

Then Mass rate of flow at section 1-1 = $\rho_1 A_1 V_1$

" " " " 2-2 = $\rho_2 A_2 V_2$

According to the principle of conservation mass Mass remains constant at all cross section.

i.e $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$

For incompressible fluids i.e liquids

$\rho_1 = \rho_2$ (Density Constant)

$\Rightarrow A_1 V_1 = A_2 V_2$

$$\boxed{AV = \text{constant}}$$

" If no fluid is added or removed from the pipe in any length then the mass passing across different sections shall be same.

Preoof

Energy in flowing liqueid.

① **Preessueree Energy.**

$$\text{Eneregy} = \text{workdone on a body}$$
$$= \text{Force} \times \text{displacement}$$
$$= \text{Preessueree} \times \text{Areea} \times \text{displacement}$$
$$= P \times V$$

Pressueree Eneregy pere unit voleume $= \dfrac{PV}{V} = P$

Preessuree Eneregy pere unit mass $= \dfrac{PV}{M} = \dfrac{P}{\rho}$

Preessueree Eneregy pere unit weight $= \dfrac{PV}{mg} = \dfrac{P}{\rho g}$

② **Kinetic Eneregy**

$$K.E = \frac{1}{2} M v^2$$

K.E. pere unit voleume $= \dfrac{\frac{1}{2} M v^2}{V} = \cancel{\dfrac{v^2}{2g}}\ \dfrac{1}{2}\rho v^2$

K.E pere unit mass $= \dfrac{\frac{1}{2} M v^2}{M} = \dfrac{1}{2} v^2$

K.E pere unit weight $= \dfrac{\frac{1}{2} M v^2}{Mg} = \dfrac{v^2}{2g}$

③ **Potential Eneregy**

$$PE = mgh$$

P.E Pere unit voleume $= \dfrac{mgh}{V} = \rho g h$

P.E pere unit mass $= \dfrac{Mgh}{M} = gh$

P.E pere unit weight $= \dfrac{Mgh}{Mg} = h$

Ans.

Considere two sections ① & ② in the pipe.

Let P_1 - Pressure at section ①
V_1 - velocity of flow at section ①
a_1 - C.S Area of section ①
h_1 - Height of the centre of the pipe from ground at section ①

Similarly P_2, V_2, a_2, h_2 are the corresponding values at section 2.

During a flow work is done by pressure to create velocity and height difference.

Energy given $= P_1 V_1 - P_2 V_2$

$V_1 = V_2 = V$ (volume remains constant as per continuity eqⁿ)

$= (P_1 - P_2)V$ (Loss in pressure energy)

Change in K.E $= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

change in Potential Energy $= mgh_2 - mgh_1$

Loss of energy = Gain in energy (according to law of conservation of energy)

$\Rightarrow$ Energy Given = Energy taken

$\Rightarrow (P_1 - P_2)V = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$

$\Rightarrow P_1 V - P_2 V = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$

$\Rightarrow P_1 V + \frac{1}{2}mv_1^2 + mgh_1 = P_2 V + \frac{1}{2}mv_2^2 + mgh_2$

Energy per unit weight

$$= \frac{P_1 V}{mg} + \frac{\frac{1}{2} m v_1^2}{mg} + \frac{mgh_1}{mg} = \frac{P_2 V}{mg} + \frac{\frac{1}{2} m v_2^2}{mg} + \frac{mgh_2}{mg}$$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + h_2$$

$$\Rightarrow \boxed{\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}}$$

<u>Application</u>

<u>Aeroplane</u>

$V_2 > V_1$

According to Bernoulli's principle

$$\frac{P_1}{\rho g} + \frac{1}{2g} v_1^2 + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$Z_1 = Z_2$ in this case

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

If $V_2 > V_1$ (if velocity increase then
$P_1 > P_2$ Pressure decrease)

Resultant force is in upward direction and aeroplane flies upward. (Lift is created upward)
Flying of aeroplane based on Bernoulli's theorem.

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Bernoulli's Equation for Real Fluid

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_L$$

where h_L = loss of energy between points 1 & 2.

Applications of Bernoulli's Equation.

The common applications on measuring devices are

- Venturimeter
- Orifice meter
- Pitot - tube

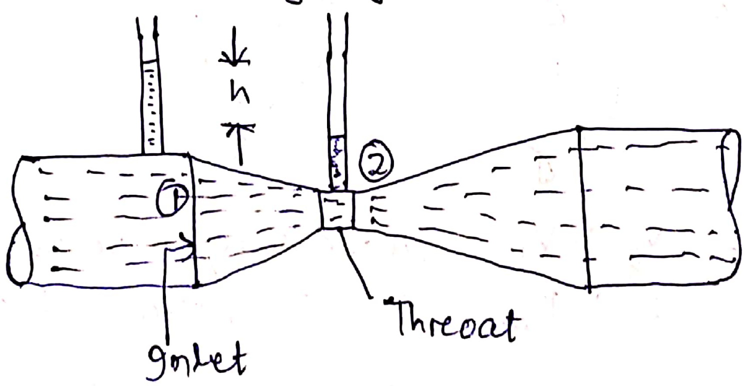
Venturimeter

A Venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts.

(i) A short converging part

(ii) Throat

(iii) Diverging part



Consider a Venturimeter fitted in a horizontal pipe through which a fluid is flowing

Let d_1 = diametere at inlet ore section ①
P_1 = preessuree at section ①
V_1 = velocity of fluid at section ①
a_1 = area at section ① $= \frac{\pi}{4} d_1^2$

Similarly d_2, P_2, V_2, a_2 are the corerespondng values at section ②

Applyng Berenoulli's Equation at section ① and ②

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

As the pipe is horeizontal $Z_1 = Z_2$

$$\Rightarrow \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} = \frac{V_2^2 - V_1^2}{2g}$$

But $\frac{P_1 - P_2}{\rho g}$ is the difference of preessure heads at section 1 and 2 and it is equal to h

$$\Rightarrow \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = h$$

According to continuity equation.

$$a_1 V_1 = a_2 V_2$$

$$V_1 = \frac{a_2 V_2}{a_1}$$

Substituting the value in above equation.

$$\Rightarrow \frac{V_2^2}{2g} - \frac{(a_2 V_2 / a_1)^2}{2g} = h$$

$$\Rightarrow \frac{V_2^2}{2g} - \frac{a_2^2 V_2^2}{a_1^2\, 2g} = h$$

$$\Rightarrow \frac{V_2^2}{2g}\left(1 - \frac{a_2^2}{a_1^2}\right) = h$$

$$\Rightarrow \frac{V_2^2}{2g}\left(\frac{a_1^2 - a_2^2}{a_1^2}\right) = h$$

$$\Rightarrow V_2^2 = 2gh\left(\frac{a_1^2}{a_1^2 - a_2^2}\right)$$

$$\Rightarrow V_2 = \frac{a_1\sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$\Rightarrow Q_{th} = a_2 V_2 = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$\Rightarrow Q_{act} = C_d \times Q_{th}$$

$$= C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

C_d = co-efficient of ventureimetere (its value is less than 1)

Value of 'h' given by differential U-tube Manometere

Case-1 Differential Manometere contains a heavy liquid.

$$h = x\left[\frac{S_h}{S_0} - 1\right]$$

x = Difference of heavy liquid column.

S_h = sp. greavity of heavy liquid

S_0 = sp. greavity of liquid flowing in pipe.

Case - II Differential manometere containing light liquid.

$$h = x\left[1 - \frac{S_L}{S_0}\right]$$

S_l = sp. greavity of light liquid.

S_0 = sp. greavity of liquid flowing in pipe.

x = Difference in light liquid coloumn.

Case - III

Inclined venturimetere having differential maonometere containing heavy liquid.

$$h = \left(\frac{P_1}{\rho g} + h_1\right) - \left(\frac{P_2}{\rho g} + h_2\right) = x\left[\frac{S_h}{S_0} - 1\right]$$

Case - IV

Inclined venturimetere having differential manometere containing light liquid.

$$h = \left(\frac{P_1}{\rho g} + h_1\right) - \left(\frac{P_2}{\rho g} + h_2\right) = x\left[1 - \frac{S_l}{S_0}\right]$$

Pitot tube

- It is a device used fore measureing the velocity of flow at any point in a pipe ore a channel.
- It is based on the preincipele that if the velocity of flow at a point becomes zero, the preessure there is increased due to converesion of K.E in to Preessure energy.
- The pitot tube consists of a glass tube bent at reight angles

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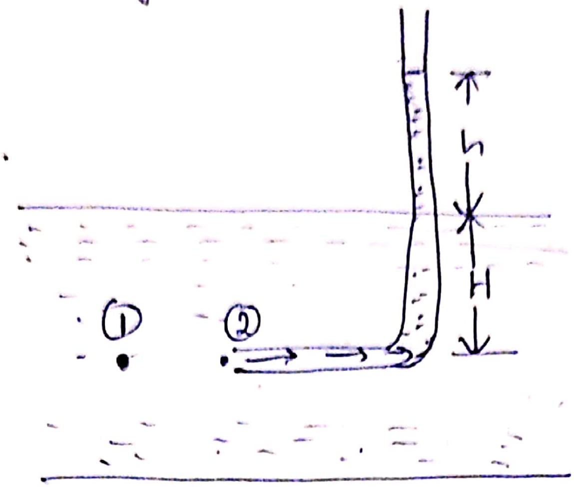
- The lower end which is bent through 90° is directed in the upstream direction.
- The liquid rises up in the tube due to the conversion of kinetic energy in to pressure energy



Consider two point ~~at~~ 1 and 2 at the same level in such a way that point 2 is just as the inlet of the pitot tube and point 1 is fare away from the tube.

P_1 = Intensity of preessuree of Point 1

V_1 - velocity of flow at 1

P_2 = preessuree at point 2

V_2 = Velocity at point 2 which is zero

H = Depth of tube in the liquid

h = reise of liquid in the tube above the free surface.

Applying Bernoulli's equation at point 1 and 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_1^2}{2g} + Z_2$$

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Two points are in a horizontal line

so $Z_1 = Z_2$

$$V_2 = 0$$

$$\frac{P_1}{\rho g} = H$$

$$\frac{P_2}{\rho g} = h + H$$

Substituting these values.

$$\Rightarrow \cancel{H} + \frac{V_1^2}{2g} = h + \cancel{H}$$

$$\Rightarrow \frac{V_1^2}{2g} = h$$

$$\Rightarrow V_1 = \sqrt{2gh}$$

Actual velocity will be

$$(V_1)_{act} = C_v\sqrt{2gh}$$

Simple Numericals

Bernoulli's Equation

Q.1 Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm2 (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section which is 5 m above datum line.

Solution

Dia of pipe = 5 cm = 0.5 m

$p = 29.43 \ N/cm^2 = 29.43 \times 10^4 \ N/m^2$

$v = 2 \ m/s$

$h = 5 \ m$

Total head or Total energy per unit weight

$$= \frac{p}{\rho g} + \frac{v^2}{2g} + h$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{2^2}{2 \times 9.81} + 5$$

$$= 30 + 0.204 + 5 = 35.204 \ m$$

Q2 A pipe through which water is flowing, is having diameters 20 cm and 10 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head of sections 1 and 2 and also rate of discharge.

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Given

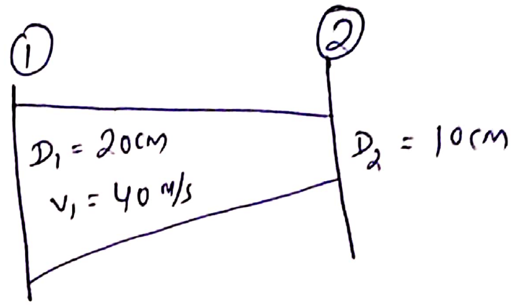
$D_1 = 20cm \Rightarrow 0.2m$

$A_1 = \pi/4\, D_1^2 = 0.0314\, m^2$

$V_1 = 4\, m/s$

$D_2 = 0.1\, m$

$A_2 = \pi/4 (0.1)^2 = 0.00785\, m^2$



i) Velocity head at section 1

$$\frac{V_1^2}{2g} = \frac{4^2}{2 \times 9.81} = 0.815m$$

ii) Velocity head at section 2

$$\frac{V_2^2}{2g} =$$

We know $A_1 V_1 = A_2 V_2$

$$\Rightarrow V_2 = \frac{A_1 V_1}{A_2} = \frac{0.0314 \times 4}{0.00785} = 16\, m/s$$

$$\frac{V_2^2}{2g} = \frac{16^2}{2 \times 9.81} = 83.047m$$

iii) Rate of discharge $= A_1 V_1$ or $A_2 V_2$

$= 0.0314 \times 4$

$= 0.1256\, m^3/s = 125.6\, lts/sec$

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CHAPTER-5

Orifices, notches & Weires

5.1 Define Orifice

5.2 Flow through Orifice

5.3 Orifices coefficient & the relation between the orifice co-efficient

5.4 Classification of notches & weires

5.5 Discharge over a rectangular notch or weir

5.6 Discharge over a triangular notch or weir

5.7 Simple problems on above.

Orifice

Definition

- Orifice is a small opening of any cross-section (such as circular, triangular, rectangular etc) on the side or at the bottom of a tank, through which a fluid is flowing.
- Orifices are used for measuring the rate of flow of fluid.

Classification

The orifices are classified on the basis of their size, shape, nature of the discharge and shape of the upstream edge.

① According to the size of Orifice
- Small orifice
- Large orifice

② According to the shape of orifice
- Circular Orifice
- Triangular Orifice
- Rectangular Orifice
- Square Orifice

③ According the nature of discharge
- Free discharging orifices
- Drowned or submerged orifice

④ According to the shape of upstream edge.
- Sharp edged orifice
- Bell mouthed orifice

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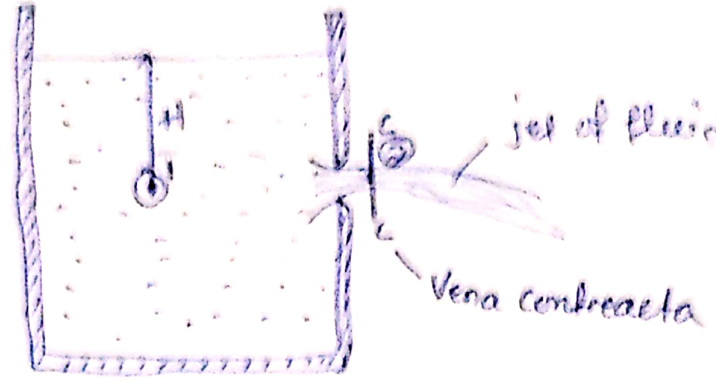
6.7 Flow through an orifice



Consider a tank fitted with a circular orifice in one of its side.

The liquid flowing through the orifice forms a jet of liquid whose area is less than the area of the orifice.

The section where the area of jet is minimum and the streamlines are straight and parallel to each other is known as <u>Vena Contracta</u>.

Consider two points 1 & 2

Point (1) is inside the tank

Point (2) is at vena contracta

Let the flow is steady and H is the head of liquid above the centre of the orifice.

Applying Bernoulli's Equation at point (1) & (2)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$Z_1 = Z_2$$

$\frac{P_1}{\rho g} = H$, $\frac{P_2}{\rho g} = 0$ $V_1 = 0$ (V_1 is very small as compared to V_2 so neglected)

Substituting all values in the eqn

$\Rightarrow$ $H + 0 = 0 + \frac{V_2^2}{2g}$ $\Rightarrow$ $V_2 = \sqrt{2gH}$

$Q = a_2 V_2 = a_2\sqrt{2gH}$

...raulic coefficients (orifice co-efficients) & their relation

The orifice co-efficients are

1. Co-efficient of velocity C_v
2. Co-efficient of contraction C_c
3. Co-efficient of discharge C_d

Co-efficient of velocity (C_v)

It is defined as the ratio between the actual velocity of a jet of liquid at vena-contracta and the theoretical velocity of jet. It is denoted by C_v.

Mathematically $C_v = \dfrac{\text{Actual velocity of jet at vena contracta}}{\text{Theoretical Velocity}}$

$$= \frac{V}{\sqrt{2gH}}$$

The value of C_v varies from 0.95 to 0.99

generally C_v is taken as 0.98.

Co-efficient of contraction (C_c).

It is defined as the ratio of the area of the jet at vena contracta to the area of the orifice. It is denoted by C_c.

Let a = area of orifice

a_c = area of jet at vena contracta

$$C_c = \frac{\text{area of jet at venacontracta}}{\text{Area of orifice}}$$

$$= \frac{a_c}{a}$$

The value of C_c varies from 0.61 to 0.69

In general the value of C_c may be taken 0.64

Co-efficient of Discharge (C_d)

It is defined as the ratio of the actual discharge from an orifice to the theoretical discharge from the orifice. It is denoted by C_d.

Let Q = actual discharge

Q_{th} = Theoretical discharge

$$C_d = \frac{Q}{Q_{th}} = \frac{\text{Actual discharge}}{\text{theoretical discharge}}$$

$$= \frac{\text{Actual Area} \times \text{Actual velocity}}{\text{Theoretical Area} \times \text{Theoretical velocity}}$$

$$= \frac{\text{Actual Area}}{\cancel{\text{Actual}}\ \text{Theoretical area}} \times \frac{\text{Actual velocity}}{\text{Theoretical velocity}}$$

$$= \frac{a_c}{a} \times \frac{V}{V_{th}}$$

$$= C_c \times C_v$$

$$\boxed{C_d = C_c \times C_v}$$

The value of C_d varies from 0.61 to 0.65. In general C_d is taken as 0.62.

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NOTCHES AND WEIRS

Notch

* A notch is a device used fore measureing the reate of flow of a liquid threough a Smale channel ore a tank.

* It may be defined as an opening in the side of a tank ore a Small channel in such a way that the liquid surface in the tank ore channel is below the top edge of the opening.

Weire

* A weire is a concreete ore masonarey Streuctuere placed in an open channel overe which the flow occures.

* It is generally in the forem of veretical wall with sharep edge at the top.

* The notch is of Small size whilee weire is of a biggere size.

* Notch is generally made of metallic plate whilee weire is made of Concreete ore Masonarey streuctuere.

Nappe ore Vein

The sheet of watere flowing threough a notch ore overe a weire is called Nappe ore Vein.

Creest ore Sill

The bottom edge of a notch ore a top of a weire overe which the watere flows known as the sill ore creest.

5.4 Classification of notches and weires

(A) The notches aree classified as

1. According to the shape of the opening
   - a) Rectangulare notch
   - b) Treiangulare notch
   - c) Treapezoidal notch
   - d) Stepped notch

2. According to the effect of the sizes on the nappe
   - a) Notch with end contreaction.
   - b) Notch without end contreaction.

(B) The Weires aree classifred as

1. According to the shape of the opening
   - a) Rectangulare weire
   - b) Treiangulare weire
   - c) Treapezoidal Weire

2. According to the shape of the creest
   - a) ~~Sharap~~ Creested Weire
   - b) Breoad creested weire
   - c) Narreow creested weire
   - d) Ogee - shaped weire.

3. According to the effect of sides on the emeregmg nappe
   - a) Weire with end contreaction
   - b) Weire without end contreaction,

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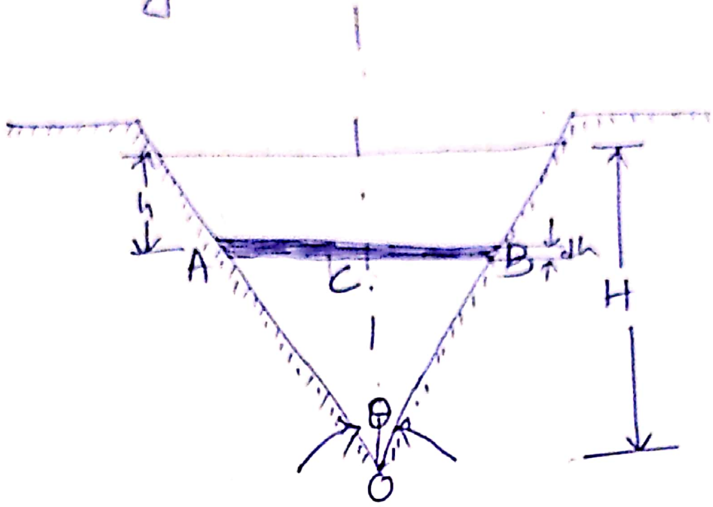
harge over a triangular notch or weir

The expression for discharge over a triangular notch or weir is the same

Let H = head of water above the V-notch

θ = angle of notch



For finding out the total discharge, consider an elementary strip of water of thickness dh at a depth h from free surface of liquid.

$$dQ = C_d \times da \times V$$

$$= C_d \times dh \times AB \times \sqrt{2gh}$$

We know $\tan\theta/2 = \frac{AC}{OC} = \frac{AC}{H-h}$

$$\Rightarrow AC = (H-h)\tan\theta/2$$

$$AB = 2 \times AC$$

$$= 2(H-h)\tan\theta/2$$

Substituting

$$dQ = C_d \times dh \times 2(H-h)\tan\theta/2 \times \sqrt{2gh}$$

$$= C_d \times 2 \times (H-h) \times \tan\theta/2 \times \sqrt{2gh} \times dh$$

$$\int_0^Q dQ = \int_0^H C_d \times 2 \times (H-h) \times \tan\theta/2 \times \sqrt{2gh} \times dh$$

~~Q = Cd×2 × H-h)~~

$$= 2 \times C_d \times \sqrt{2g} \tan\theta/2 \int_0^H (H-h) \times \sqrt{h} \times dh$$

$$= 2 \times C_d \times \sqrt{2g} \times \tan\theta/2 \int_0^H (Hh^{1/2} - h^{3/2}) dh$$

$$= 2 \times C_d \times \sqrt{2g} \times \tan\theta/2 \left[\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2} \right]_0^H$$

$$= 2 \times C_d \times \sqrt{2g} \times \tan\theta/2 \left[\frac{H \times H^{3/2}}{3/2} - \frac{H^{5/2}}{5/2} \right]$$

$$= 2 \times C_d \times \sqrt{2g} \times \tan\theta/2 \left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2} \right]$$

$$= 2 \times C_d \times \sqrt{2g} \times \tan\theta/2 \times H^{5/2} \times \frac{4}{15}$$

$$= \frac{8}{15} \times C_d \times \sqrt{2g} \times \tan\theta/2 \times H^{5/2}$$

$$\boxed{Q = \frac{8}{15} \times C_d \times \sqrt{2g} \times \tan\theta/2 \times H^{5/2}}$$

5.6

Discharege overe a reectangulare notch ore weire

The expression fore discharege overe a reectangulare notch ore weire is the same.

Considere a reectangulare notch ore weire preovided in a Channel carereying watere.

Let H = Head of watere overe the creest

L = Length of the notch ore weire.

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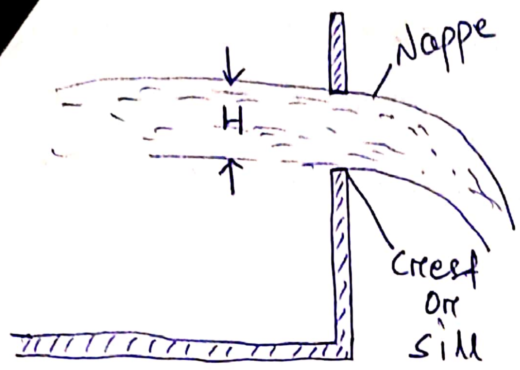

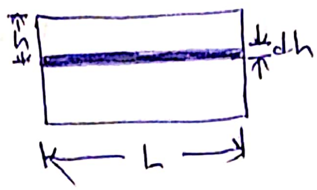

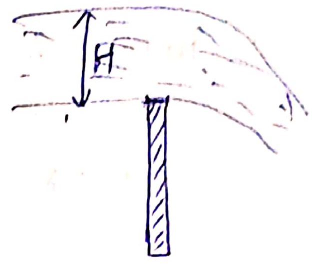

rectangular notch

Rectangular weir

Consider an elementary horizontal strip of water of thickness dh and length L at a depth h from the free surface of water.

The area of the strip $= L \times dh$

Theoretical velocity of water flowing through the strip $= \sqrt{2gh}$

discharge through the strip is

$$dQ = C_d \times dA \times V$$

$$= C_d \times L \times dh \times \sqrt{2gh}$$

Total discharge $Q = \int_0^Q dQ$

$$\Rightarrow \int_0^Q dQ = \int_0^H C_d \times L \times \sqrt{2gh} \times dh$$

$$= C_d \times L \times \sqrt{2g} \int_0^H h^{1/2} dh$$

$$= C_d \times L \times \sqrt{2g} \left[\frac{h^{3/2}}{3/2} \right]_0^H$$

$$= C_d \times L \times \sqrt{2g} \times \frac{2}{3} \times H^{3/2}$$

$$\boxed{Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}}$$

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Nuemereicals

Q.1 The head of watere overe an oreifice of diametere 4[illegible] is 10 m. Find the actual discharege and actual velocity of the jet at vena contracta. Take $C_d = 0.6$ and $C_v = 0.98$

Solution

Given data

$H = 10\,m$

$d = 40\,mm = 0.04\,m$

$a = \pi/4\, d^2 = \pi/4 \times (0.04)^2 = 0.001256\, m^2$

$C_d = 0.6$

$C_v = 0.98$

Actual velocity $= C_v \sqrt{2gH}$

$= 0.98 \sqrt{2 \times 9.81 \times 10}$

$= 13.72\; m/s$

Actual Discharege $= C_d \times Q_{th}$

$= C_d \times (a_{th} \times V_{th})$

$= 0.6 \times 0.001256 \times \sqrt{2 \times 9.81 \times 10}$

$= 0.01054\; m^3/s$

Q.2 The head of watere overe the centree of an orifice of diametere 20 mm is 1 m. The actual discharege through the oreifice is 0.85 lt/s. Find the Co-efficient of discharge.

Solution

Given data

$H = 1\,m$

$d = 20\,mm = 0.02\,m$

$a = \pi/4\, d^2 = \pi/4\, (0.02)^2 = 0.000314\, m^2$

$Q_{act} = 0.85\, lt = 0.85 \times 10^{-3}\; m^3/sec$

$$Q_{th} = a_{th} \times V_{th}$$

$$= \frac{\pi}{4} d^2 \times \sqrt{2gH}$$

$$= 0.000314 m^2 \times \sqrt{2 \times 9.81 \times 1}$$

$$= 0.00139 \ m^3/sec$$

$$C_d = \frac{Q_{act}}{Q_{th}} = \frac{0.00085}{0.00139} = 0.61 \quad \text{Ans}$$

Q3 Find the discharge over a triangular notch of angle 60° when the head over the V-notch is 0.3m Assume $C_d = 0.6$.

Solution

Given data

$\theta = 60^\circ$

$C_d = 0.6$

$H = 0.3M$

$$Q = \frac{8}{15} \times C_d \times \tan\frac{\theta}{2} \times \sqrt{2g} \times H^{5/2}$$

$$= \frac{8}{15} \times 0.6 \times \tan 30^\circ \times \sqrt{2 \times 9.81} \times (0.3)^{5/2}$$

$$= 0.040 \ M^3/s \quad \text{Ans}$$

Q.4 Find the discharge of water flowing over a rectangular notch of 2m length when the constant head over the notch is 300 MM. Take $C_d = 0.60$.

Solution

$L = 2M$

$H = 300 \ mm = 0.3 \ M$

$C_d = 0.60$

$$Q = \frac{2}{3} \times C_d \times L \times \sqrt{2g} \times H^{3/2}$$

$$= \frac{2}{3} \times 0.6 \times 2.0 \times \sqrt{2 \times 9.81} \times (0.3)^{3/2}$$

$$= 0.582 \ m^3/s$$

Ans

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CHAPTER-6

FLOW THROUGH PIPE

6.1 Definition of Pipe.

6.2 Loss of energy in pipes

6.3 Head loss due to friction: Darcy's and Chezy's formula (Expression only)

6.4 Solve problems using Darcy's and Chezy's formula

6.5 Hydraulic Gradient and Total Energy Line

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Definition of Pipe

Pipe is defined as a close tube used to convey water, gas, oil or other fluid substances.

6.2 Loss of energy in pipes

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost.

This loss of energy is classified as

Energy Losses

- **Major Energy Losses**
  - This is due to friction and it is calculated by the following formulae
    - a) Darcy-Weisbatch formula
    - b) Chezy's formula
- **Minor Energy Losses**
  - This is due to
    - a) Sudden expansion of pipe
    - b) Sudden contraction of pipe
    - c) Bend in pipe
    - d) Pipe fittings etc
    - e) An obstruction in pipe.

6.3 Head Loss due to friction

① Darcy-Weisbatch Formula:

According to Darcy-Weisbach the loss of head or energy due to friction can be calculated by using the following formula.

$$\boxed{h_f = \frac{4fLV^2}{2gd}}$$

where h_f = head loss due to friction

f = co-efficient of friction

$= \frac{16}{R_e}$ (if $R_e < 2000$) $\left(R_e = \frac{Vd}{\nu}\right)$

$= \frac{0.079}{R_e^{1/4}}$ when R_e lies between 4000 to 10^6

L = Length of the pipe

V = Mean velocity of flow

d = diameter of pipe.

Chezy's formula

According to chezy

$$V = C\sqrt{mi}$$

where V = Mean velocity of flow

C = Chezy's constant

m = Hydraulic mean depth

$= \frac{A}{P} = \frac{\text{Area of flow}}{\text{Perimeter}}$

$= \frac{\frac{\pi}{4}d^2}{\pi d} = \frac{d}{4}$

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Hydraulic Gradient and Total Energy Line

① Hydraulic Gradient Line (HGL)

It is defined as the line which gives the sum of Pressure head $\left(\frac{P}{\rho g}\right)$ and datum head (z) of a flowing fluid in pipe with respect to some reference line.

Or

It is the line which is obtained by joining the top of all vertical ordinates showing pressure head of a flowing fluid in a pipe from the centre of the pipe.

② Total Energy Line (TEL)

It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line.

Or

It is defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of Pressure head and kinetic head from the centre of the pipe.

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CHAPTER-7 Impact of Jet

7.1 Impact of jet on fixed and moving veretical flat plates.

7.2 Dereivation of work done on sereies of vanes and condition fore maximum efficiency.

7.3 Impact of jet on moving curved vanes, illustration using velocity treiangles, dereivation of work done, efficiency.

Impact of jet

Jet : It is a stream of fluid issuing from a nozzle with a high velocity.

Impact of jet : Impact of jet means the force exerted by the jet on a plate which may be stationary or moving.

The various cases of Impact of jet are

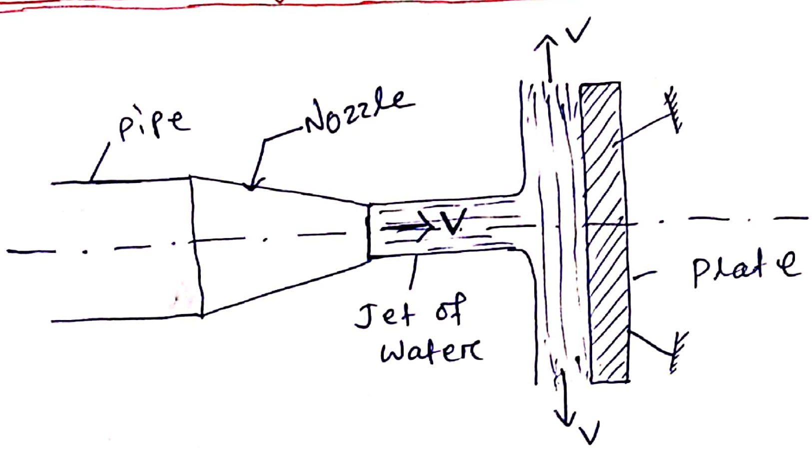
ⓐ Force exerted by the jet on stationary plate when

1. Plate is vertical to jet
2. plate is inclined to jet
3. plate is curved.

ⓑ Force exerted by the jet on a moving plate when

1. plate is vertical to jet
2. plate is inclined to the jet
3. plate is curved.

Force Exerted By the jet on a Fixed vertical plate.



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Considere a jet of watere coming out freom the no... strikes a flat vertical plate

Let V = velocity of jet

d = diametere of the jet

a = areea of creoss-section of the jet

$= \frac{\pi}{4} d^2$

The jet after striking will get deflected through 90° as the plate is fixed. Hence the component of jet in the direction of jet aftere striking will be zero.

The force exerted by the jet on the plate in the direction of jet

F_x = Rate of change of momentum in the direction of force.

$$= \frac{\text{Initial momentum} - \text{Final momentum}}{\text{Time}}$$

$$= \frac{\text{Mass} \times \text{Initial velocity} - \text{Mass} \times \text{Final velocity}}{\text{Time}}$$

$$= \frac{\text{Mass}}{\text{Time}} \left[\text{Initial velocity} - \text{Final velocity}\right]$$

$$= \frac{\text{Mass}}{\text{Time}} \left[\text{velocity of jet before striking} - \text{velocity of jet after striking}\right]$$

$$= \rho a V \left[V - 0\right]$$

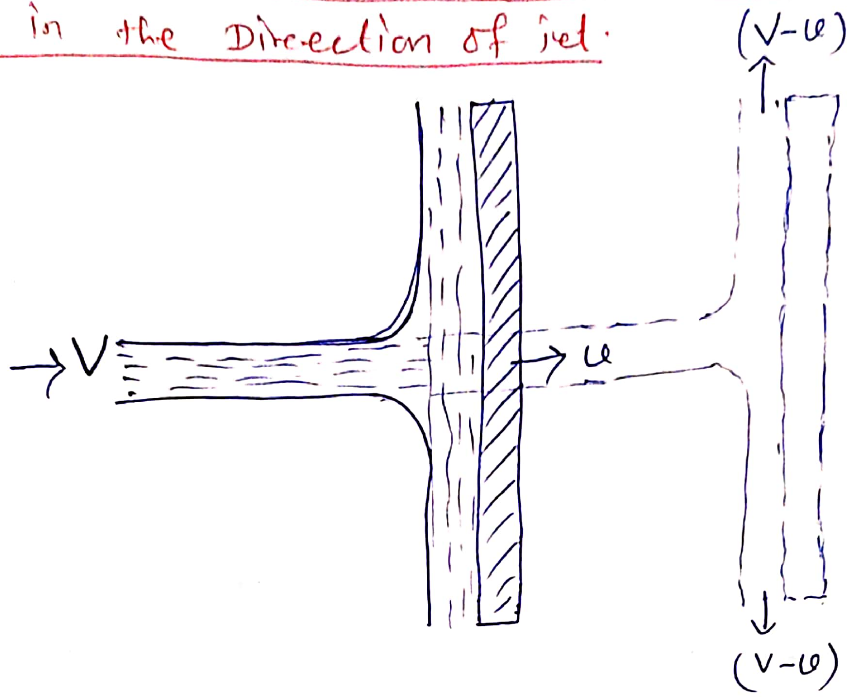
$$= \rho a V^2$$

$$\left[\ \rho = \frac{M}{V} \Rightarrow M = \rho V \Rightarrow M/\text{sec} = \rho V/\text{sec}\right.$$

$$V/\text{sec} = Q = a \times V$$

$$\Rightarrow M/\text{sec} = \rho a V$$

Force exerted by a jet on a flat vertical plate moving in the direction of jet.



Consider a jet of water striking a flat vertical plate moving with a uniform velocity away from the jet.

Let V = velocity of jet

u = velocity of flat plate

a = area of the cross-section of the jet

The jet strikes the plate with a relative velocity $(V-u)$

Hence $V_r = V-u$

Mass of water striking the plate per sec

$= \rho \times a \times$ velocity with which the jet strikes the plate

$= \rho a(V-u)$

Force exerted by the jet on the plate in the direction of jet

F_x = Mass/sec [Initial velocity with which water strikes − Final velocity]

$= \rho a(V-u)\,[(V-u)-0]$

$= \rho a(V-u)^2$

In this case work done by the jet on the plate as the plate is moving

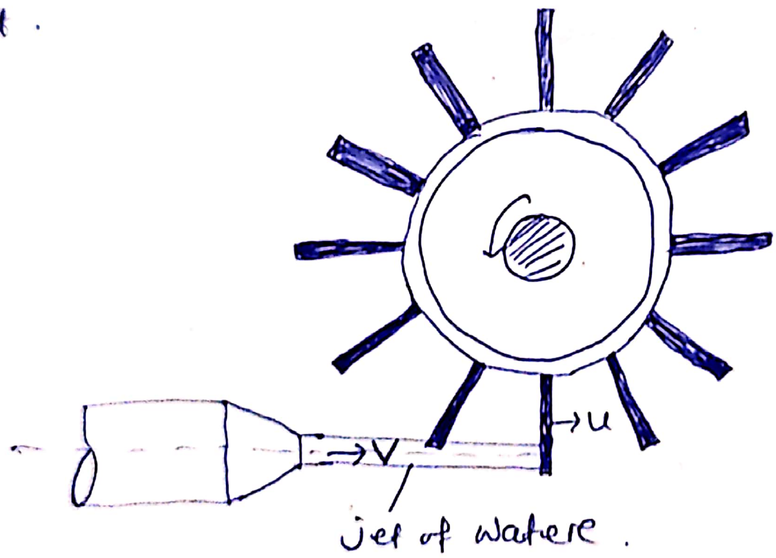
$$\text{Work done/sec} = \text{Force} \times \text{Distance/sec}$$

$$= F_x \times u$$

$$= \rho a (V-u)^2 \times u \quad \text{Nm/s or watt.}$$

7.2 Derivation of work done on series of flat plates and condition for maximum efficiency.

The force exerted by a jet of water on a single moving plate is not practically feasible. Actually a large number of plates are mounted on the circumference of a wheel at a fixed distance apart.



Jet striking a series of vanes.

Let V = velocity of jet

d = Diameter of jet

a = Cross-sectional area of jet

$= \frac{\pi}{4} d^2$

u = velocity of vane

In this case the mass of water coming out from the nozzle per second is always in contact with the plate when all plates are considered.

Mass of water striking per second = $\rho a V$

Jet strikes the plate with a velocity = $V - u$

$$F_x = \text{Mass/second}\,[\text{Initial velocity} - \text{final velocity}]$$

$$= \rho a V\,[(V-u) - 0]$$

$$= \rho a V (V-u)$$

$$\text{work done/sec} = F_x \times u$$

$$= \rho a V (V-u) \times u$$

Kinetic energy of the jet per second (Input energy)

$$= \frac{1}{2} m V^2$$

$$= \frac{1}{2} \rho a V \times V^2$$

$$= \frac{1}{2} \rho a V^3$$

$$\eta = \frac{\text{work done/sec}}{\text{KE/sec}}$$

$$= \frac{\rho a V (V-u) u}{\frac{1}{2} \rho a V^3} = \frac{2u(V-u)}{V^2}$$

<u>Condition for maximum efficiency.</u>

Efficiency will be maximum when

$$\frac{d\eta}{du} = 0 \Rightarrow \frac{d}{du}\left[\frac{2u(V-u)}{V^2}\right] = 0$$

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$$\Rightarrow \frac{d}{du}\left[2uV - 2u^2\right] = 0$$

$$\Rightarrow \frac{d}{du}2uV - \frac{d}{du}(2u^2) = 0$$

$$\Rightarrow 2V - 4u = 0$$

$$\Rightarrow \boxed{V = 2u}$$

Maximum Efficiency

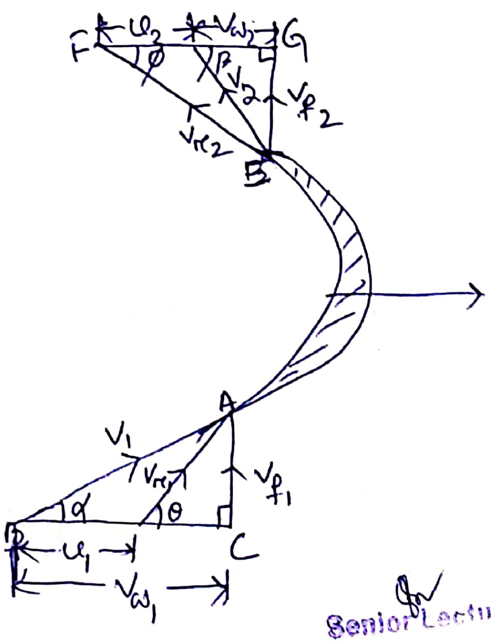
Substituting the value $V = 2u$

$$\eta_{max} = \frac{2u(2u-u)}{4u^2}$$

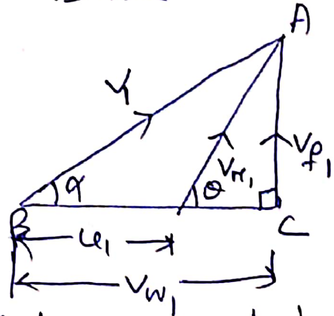
$$= \frac{2u^2}{4u^2} = \frac{1}{2}$$

$$= 50\ \%.$$

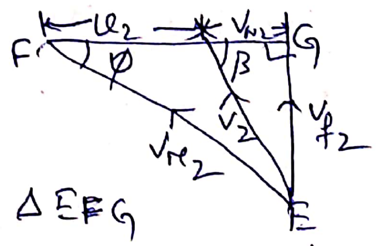
7.3 Impact of Jet on moving curved vanes, illustration using velocity triangles, derivation of workdone and efficiency.



ΔABC



Inlet velocity triangle



ΔEFG

Outlet velocity triangle

Let V_1 = velocity of jet at inlet

u_1 = velocity of plate (vane) at inlet

V_{r_1} = Relative velocity of jet and plate at inlet

α = Angle between direction of jet and direction of motion of plate or guide blade angle at inlet

θ = Vane angle at inlet

V_{w_1} and V_{f_1} = components of V_1

V_{w_1} = whirl velocity at inlet

V_{f_1} = Velocity of flow at inlet.

V_2 = velocity of jet leaving the vane

u_2 = velocity of vane at outlet.

V_{r_2} = Relative velocity of jet and plate at outlet.

β = guide blade angle at outlet

ϕ = vane angle at outlet.

V_{w_2}, V_{f_2} = components of the velocity V_2 in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet

V_{w_2} = velocity of whirl at outlet

V_{f_2} = velocity of flow at outlet.

If the vane is smooth and is having velocity in the direction of motion of vane at inlet and outlet equal.

$$u_1 = u_2 = u$$

$$V_{r_1} = V_{r_2}$$

Mass of water striking per second

$= \rho a V_{r_1}$

Force exerted by the jet in the direction of motion.

$$F_x = \frac{\text{Mass}}{\text{sec}} \left[\text{Initial velocity} - \text{Final velocity} \right]$$

Initial velocity with which the jet strikes $= V_{r_1}$

Component of V_{r_1} in the direction of motion.

$$= V_{r_1} \cos\theta = V_{w_1} - u_1$$

Final velocity $= -V_{r_2} \cos\phi$

$$= -\left[u_2 + V_{w_2} \right]$$

$$F_x = \rho a V_{r_1} \left[V_{w_1} - u_1 - \{ -(u_2 + V_{w_2}) \} \right]$$

$$= \rho a V_{r_1} \left[V_{w_1} - u_1 + u_2 + V_{w_2} \right]$$

$$= \rho a V_{r_1} \left[V_{w_1} + V_{w_2} \right] \quad (\because u_1 = u_2)$$

$$\boxed{F_x = \rho a V_{r_1} \left[V_{w_1} + V_{w_2} \right]}$$ — when β is acute

If $\beta = 90^\circ$

$$V_{w_2} = 0$$

$$F_x = \rho a V_{r_1} \left[V_{w_1} \right]$$

$\beta > 90^\circ$ (obtuse)

$$F_x = \rho a V_{r_1} \left[V_{w_1} - V_{w_2} \right]$$

In general $F_x = \rho a V_{r_1} \left[V_{w_1} \pm V_{w_2} \right]$

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$$\text{Work done/sec} = F_x \times u$$

$$= \rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u$$

Work done/second / unit weight/second

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times u}{\text{weight of fluid striking/s}}$$

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u}{g \times \rho a V_{r_1}}$$

$$= \frac{1}{g} [V_{w_1} \pm V_{w_2}] \times u \quad Nm/N$$

Workdone / second / unit mass /second.

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u}{\text{Mass of fluid striking/second}}$$

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] \times u}{\rho a V_{r_1}}$$

$$= [V_{w_1} \pm V_{w_2}] \times u \quad Nm/kg$$

Efficiency of jet

$$\eta = \frac{\text{output}}{\text{Input}} = \frac{\text{Work done per second}}{\text{Initial K.E per second}}.$$

$$= \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u}{\frac{1}{2} M V_1^2}$$

$$M = \text{mass of fluid/sec} = \rho a V_1$$

$$\Rightarrow \eta = \frac{\rho a V_{r_1} [V_{w_1} \pm V_{w_2}] u}{\frac{1}{2} \times \rho a V_1^3.}$$

Problems

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Q.1 Watere is flowing threrough a pipe at the end of which a nozzle is fitted. The diametere of the nozzle is 100 mm and the head of watere at the centree of nozzle is 100 m. Find the force exereted by the jet of watere on a fixed verctical plate. The co-efficient of velocity is given as 0.95.

Solution Given data

$d = 100\,mm = 0.1\,m$

$H = 100\,m$

$C_v = 0.95$

$a = \frac{\pi}{4}d^2 = 0.007854\ m^2$

$V = C_v\sqrt{2gH}$

$= 0.95 \times \sqrt{2 \times 9.81 \times 100}$

$= 44.294\ m/s \times 0.95 = 42.08$

$\cancel{Ra} =$ Actu.

$F = \rho a v^2$

$= 1000 \times 0.007854 \times (42.084)^2$

$= 13907.2\ N$

$= 13.9\ kN$

Q.2 A jet of watere of diameter 10 cm streikes a flat plate noremally with a velocity of 15 m/s. The plate is moving with a velocity of 6 m/s in the direction of the jet and away from the jet. Find

(i) The force exereted by the jet on the plate

(ii) Work done by the jet on the plate perr second.

Also calculate the power and efficiency of the jet

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Given data

Diametere of the jet $d = 10cm$
$= 0.1m$

$$a = \pi/4 d^2 = \pi/4 \times (0.1)^2$$
$$= 0.007854 \, m^2$$

$$V = 15 \, m/s$$
$$u = 6 \, m/s$$

(i) $F_x = \rho a (V-u)^2$

$$= 1000 \times 0.007854 \times (15-6)^2$$
$$= 636.17 \, N$$

(ii) Work done per second

$$= F_x \times u$$
$$= 636.17 \times 6$$
$$= 3817.02 \quad Nm/s \text{ or watt}$$

(iii) Powere of the jet

Powere = workdone/sec $= 3817.02$ Watt
$= 3.817$ kW

(iv) Efficiency of the jet

$$= \frac{\text{output}}{\text{Input}}$$

Output = workdone/sec $= 3817.02$ W

Input power = KE of jet/s

$$= \frac{1}{2}(\text{mass/sec}) \times V^2 = \frac{1}{2} \rho a V \times V^2$$
$$= \frac{1}{2} \times 1000 \times 0.007854 \times (15)^3 = 13253.6 \, Nm/s$$

$$\eta = \frac{3817.02}{13253.6} = 0.288 = 28.8\%$$

Q.3 A nozzle of 50 mm diameter delivers a stream of water at 20 m/s perpendicular to a plate that moves away from the jet at 5 m/s. Find:

a) The force on the plate
b) The work done
c) The efficiency of jet

Solution

$$d = 50\,mm = 0.05\,m$$

$$a = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (0.05)^2$$
$$= 0.0019635\ m^2$$

$$V = 20\ m/s$$
$$u = 5\ m/s$$

a) $$F_x = \rho a (V-u)^2$$
$$= 1000 \times 0.0019635 \times (20-5)^2$$
$$= 441.78\ N$$

b) $$\text{work done/sec} = F_x \times u$$
$$= 441.78 \times 5 = 2208.9\ W$$

c) The efficiency of the jet

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{work done by jet/sec}}{\text{K.E of the jet/sec}}$$

$$\text{work done/sec} = 2208.9\ W$$

$$\text{K.E/sec} = \frac{1}{2} m V^2 = \frac{1}{2} (\rho a V) \times V^2 = \frac{1}{2} \rho a V^3$$
$$= \frac{1}{2} \times 1000 \times 0.0019635 \times (20)^3 = 6540\ W$$

$$\eta = \frac{2208.9}{6540} = 0.3377 = 33.77\%\ \text{Ans}$$

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Q.4 A jet of water having a velocity of 20 m/s strikes a curved vane, which is moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of motion of vane at outlet. Calculate

(i) vane angles; so that the water enters and leaves the vane without shock.

(ii) work done per second per unit weight of water striking (or work done per unit weight of water striking) the vane per second.

<u>Solution</u> :

Given data

$V_1 = 20 \text{ m/s}$

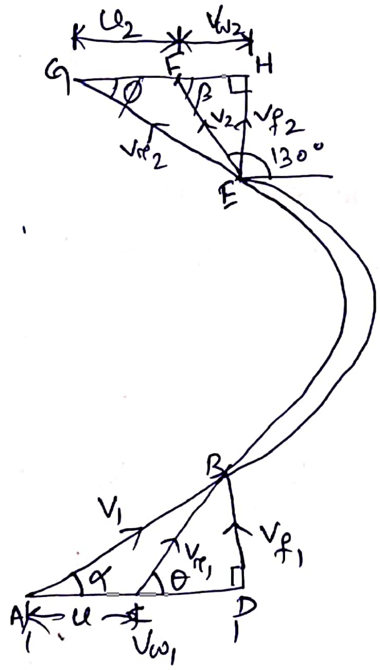
$u_1 = u_2 = u = 10 \text{ m/s}$

$\alpha = 20°$

$\beta = 180° - 130°$

$= 50°$

$V_{r_1} = V_{r_2}$



<u>To Find</u>

(i) θ & ϕ

(ii) Work done/sec/unit wt/sec.

(i) From inlet velocity triangle ABD

$$V_{w_1} = V_1 \cos\alpha = 20 \times \cos 20° = 18.794 \text{ m/s}$$

$$V_{f_1} = V_1 \sin\alpha = 20 \times \sin 20° = 6.84 \text{ m/s}$$

$$\tan\theta = \frac{BD}{CD} = \frac{V_{f_1}}{V_{w_1} - u} = \frac{6.84}{18.794 - 10}$$

$$\theta = 37° 52.5'$$

$V_{re_1} = V_{re_2}$

$\sin\theta = \frac{V_{f_1}}{V_{re_1}}$

$\Rightarrow V_{re_1} = \frac{V_{f_1}}{\sin\theta} = \frac{6.84}{\sin 37.875^\circ} = 11.14 \text{ m/s}$

$V_{re_1} = V_{re_2} = 11.14 \text{ m/s}$

From outlet velocity triangle EGH

Applying sine rule.

$$\frac{V_{re_2}}{\sin(180^\circ - \beta)} = \frac{u}{\sin(\beta - \phi)}$$

$$\Rightarrow \frac{11.14}{\sin(180^\circ - 50)} = \frac{10}{\sin[50^\circ - \phi]}$$

$$\Rightarrow 50 - \phi = 43.44^\circ$$

$$\phi = 6.56^\circ$$

(ii) Work done per second per unit weight of the water striking the vane per second is given by

$$= \frac{1}{g}[V_{w_1} + V_{w_2}] \times u \quad (\beta < 90^\circ)$$

$$V_{w_2} = V_{re_2}\cos\phi - u$$

$$= 11.14 \cos 6.56^\circ - 10$$

$$= 20.24 \text{ Nm/N} \quad \text{Ans}$$

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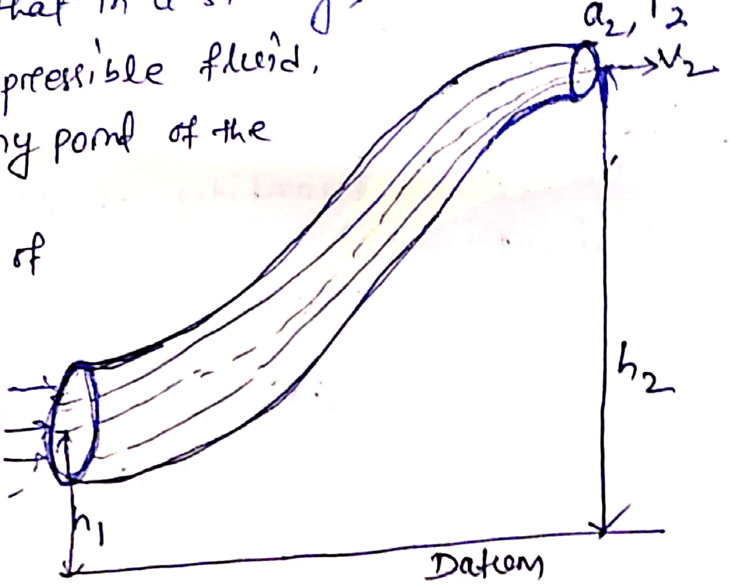
Bernoulli's Theorem

Statement: It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant.



The total energy consists of
Pressure energy,
kinetic energy, and
potential energy.

These energies per unit weight of the fluid are

$$PE = \frac{P}{\rho g}$$

$$KE = \frac{V^2}{2g}$$

$$PE = Z$$

Mathematically Bernoulli's theorem is written as

$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{Constant}$$

Assumptions

The Assumptions are
1. The fluid is ideal i.e viscosity is zero
2. The flow is steady
3. The flow is incompressible
4. The flow is irrotational.